1. Calculate all critical points of \( f(x, y) = 2x^3 - 6x - 6y^2 + y^3 \) and classify these.

Show all major steps. Check your answers.

2. Use Lagrange multipliers to calculate the minimum of \( f(x, y) = x^2 + 3y^2 \) subject to the constraint \( y = 4 - x \).

To check your work pictorially, sketch a contour diagram for \( f(x, y) \) and overlay the graph of the constraint.

Show all major steps.

3. Consider the contour diagram of a function \( z = f(x, y) \) given on the right. Selected contours are labeled on the right edge.

a. Read off the value of \( f(1, 1) \).

b. Estimate the values of \( \frac{\partial f}{\partial x}(1, 1) \) and \( \frac{\partial f}{\partial y}(1, 1) \).

c. Estimate the gradient of \( f \) at \( (x, y) = (1, 1) \).

d. Find a formula for a linear approximation of \( f \) about \( (x, y) = (1, 1) \) ("approximate tangent plane")

e. Is \( \frac{\partial^2 f}{\partial x^2}(1, 1) \) positive, zero or negative?

What about \( \frac{\partial^2 f}{\partial y^2}(1, 1) \)? EXPLAIN!

f. Find very crude upper and lower estimates for the iterated integral \( \int_{y=1}^{2} \int_{x=1}^{3} (x, y) \, dx \, dy \).

g. Briefly explain how one may get better estimates.

4. In this problem consider the vector field \( \vec{F}(x, y) = (x^2 + y^2)\hat{i} + 2xy\hat{j} \)

a. Calculate the scalar curl \( \left( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \). Is \( \vec{F}(x, y) \) a gradient vector field?

b. Find a potential function for \( \vec{F}(x, y) \). If impossible explain why.

c. Use the fundamental theorem for line integrals to find the value of the line integral \( \int_C \vec{F} \cdot d\vec{R} \) where \( C \) is the unit circle \( x^2 + y^2 = 1 \) traversed counterclockwise (starting and ending at the point (1,0)).

d. Use parameterizations to again calculate the line integral of part c. You only need to show the major steps.

5. Consider the (normalized) gravitational vector field \( \vec{G}(x, y, z) = M \frac{-x\hat{i} - y\hat{j} - z\hat{k}}{(x^2+y^2+z^2)^{3/2}} \) of a point mass \( M \) at the origin.

a. Calculate \( \frac{\partial}{\partial x} \left( \frac{x}{(x^2+y^2+z^2)^{3/2}} \right) \) by hand.

b. Calculate the curl and the divergence of \( \vec{G} \).

c. Explain why one cannot use the divergence theorem to argue that the flux integral in part b. is zero.

d. Use the divergence theorem (and parts a. and b.) to calculate the total flux \( \int_K \vec{G} \cdot d\vec{A} \) of \( \vec{G} \) through the cube \( K \) with corners \( (\pm 10, \pm 10, \pm 10) \) (oriented "outward"). Very carefully explain your reasoning.

6. a. Give the definition of "path independence" for a vector field.

b. Using the fundamental theorem for line integrals, explain why every "gradient field" is path-independent.