1. Explain the meaning of “conservative”, “gradient field”, “potential function”, “circulation”, and the relations between these terms.

<table>
<thead>
<tr>
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<th>1.5</th>
<th>2.5</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
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<td>d^2</td>
<td>d^2</td>
<td>d^2</td>
</tr>
<tr>
<td>2</td>
<td>d^2</td>
<td>d^2</td>
<td>d^2</td>
<td>d^2</td>
</tr>
<tr>
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<td>d^2</td>
<td>d^2</td>
<td>d^2</td>
<td>d^2</td>
</tr>
<tr>
<td>4</td>
<td>d^2</td>
<td>d^2</td>
<td>d^2</td>
<td>d^2</td>
</tr>
</tbody>
</table>

2a. Using the table for \( \vec{F}(x, y) \), estimate the value of the line integral \( \int_{C_1} \vec{F} \cdot d\vec{r} \) where \( C_1 \) is the rectilinear path that starts at (0, 0), and goes via (0, 3) to (3, 3).

2b. Using only the table, estimate the value of the line integral \( \int_{C_2} \vec{F} \cdot d\vec{r} \) where \( C_2 \) is the straight line segment from (0, 0) to (3, 3).

2c. Based on the results from a. and b. do you think the table represents a conservative vector field? Explain.

2d. **Bonus credit**: If possible describe a path \( C \) from (0, 0) to (3, 3) such that \( \int_{C_1} \vec{F} \cdot d\vec{r} < 0 \). If impossible explain why.

3a. If possible, describe what each pictured vector field may represent in a practical application / physical setting.

b. Which of these fields appear to be gradient fields. Explain why, if yes; why not, else.

c. Which of the fields appear to be divergence free? Explain why, if yes; why not, else.

d. Find a possible formula for each vector field pictured.

e. Find a possible formula for a potential function for each of the vector field pictured, provided it exists.

4. Calculate the line integral \( \int_{C} \vec{F} \cdot d\vec{r} \) where \( \vec{F}(x, y) = y\hat{i} \) and \( C \) is the curve formed by the edges of the triangle with corners (0, 0), (3, 3) and (0, 3) traversed counter clockwise.

5a. Calculate the total flux \( \oint_{C} \vec{E} \cdot \hat{N} \) of \( \vec{E}(x, y) = \frac{r^2}{x^2+y^2} \hat{i} \) across the circle \( C \) with center (0, 0) and radius \( r = 1 \).

b. Calculate the divergence (div\( \vec{E} \))(x, y) = \( \frac{r}{x^2+y^2} \), \( \frac{r^2}{2x^2+y^2} \), \( \frac{r^2}{x^2+2y^2} \).

c. Use your results from a. and b. and Green’s theorem in divergence form to calculate the total flux \( \oint_{C} \vec{E} \cdot \hat{N} \) of \( \vec{E}(x, y) \) across the square \( C \) with corners (±3, ±3).

6. **(Bonus credit)**: For the linear vector field \( \vec{L}(x, y) = (ax + by)\hat{i} + (cx + dy)\hat{j} \) calculate the line integral \( \int_{C} \vec{F} \cdot d\vec{r} \) where \( C \) is the triangle with corners \((x_0, y_0)\), \((x_0 + \Delta x, y_0)\), and \((x_0, y_0 + \Delta y)\) (traversed counter-clockwise).

b. Comment on the significance of the result.

7. **(Bonus credit)**: Explain the meaning of the symbols, the hypotheses these objects must satisfy, and their relation to each other; explain each step in the following string of (approximate) equations. Explain what will happen in the limit (what kind of limit?).

\[
\int \int_R \text{div} \vec{F} \, dA \approx \sum_i \int \text{div} \vec{F}(p_i) \Delta A_i \approx \sum_i \frac{\int_{C_i} \vec{F} \cdot \hat{N} \, ds}{\Delta A_i} \Delta A_i = \sum_i \int_{C_i} \vec{F} \cdot \hat{N} \, ds = \int_{C} \vec{F} \cdot \hat{N} \, ds
\]