Show your work - explain what you are doing.
It is YOUR responsibility to demonstrate that you have mastered the material of this class.

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You may bring in one page of formulas – which you have to turn it in with the test.

1. a. Find all critical points of the function \( f(x, y) = x^3 - 3x^2 - y^3 + 3y \).
b. Determine at which a local maximum, a local minimum or a saddle point occurs.
c. Show all your work, and explain your reasoning step by step!

2. Consider the function \( f(x, y) = xy^2 \).
a. Sketch several cross-sections of the graph parallel to the \( yz \)-plane. Explain.
b. Sketch several cross-sections of the graph parallel to the \( xz \)-plane. Explain.
c. Describe the shape of the graph in words.
d. Find the equation of the tangent plane to the graph of \( z = f(x, y) \) at \( (x, y) = (0, 0) \),
e. Find the second order Taylor approximation for \( f(x, y) \) at \( (x, y) = (0, 0) \).

3. Consider the table of function values for a function \( z = f(x, y) \) on the left. (\( x \)-values listed across, \( y \)-values up-down.)
a. Sketch a contour diagram.
b. Describe the shape of the graph in words.
c. Sketch the graphs for cross-sections parallel to the \( xz \)-plane for various values of \( y \).
d. Estimate the following derivatives:
\[
\frac{\partial f}{\partial x}(1, 2), \quad \frac{\partial f}{\partial y}(1, 2),
\]
e. Is \( \frac{\partial^2 f}{\partial x^2}(1, 2) \) positive, negative or zero? Explain why!

4. a. Calculate the curvature \( \kappa(t) \) for the curve \( (x, y) = (t \cos(t), t \sin(t)) \) as a function of time.
b. Sketch the graph of the curvature as a function of time for \( t > 0 \).
c. Comment in one or two sentences how this graph agrees with your expectations.

5. For the “coordinate change” \( x = r \cos(\Theta), y = r \sin(\Theta) \), with “inverse” \( r = \sqrt{x^2 + y^2}, \Theta = \arctan(\frac{y}{x}) \),
a. calculate the total derivatives, that is, the matrices of partial derivatives
\[
A = \begin{pmatrix}
\frac{\partial x}{\partial r} & \frac{\partial x}{\partial \Theta} \\
\frac{\partial y}{\partial r} & \frac{\partial y}{\partial \Theta}
\end{pmatrix}
\quad \text{and} \quad
B = \begin{pmatrix}
\frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\
\frac{\partial \Theta}{\partial x} & \frac{\partial \Theta}{\partial y}
\end{pmatrix}
\] 

b. Calculate the determinant of the matrix \( A \) (simplify the result).
c. Is it true that \( \frac{\partial x}{\partial r} \cdot \frac{\partial r}{\partial x} = 1 \) ?

**Bonus**: Calculate the determinant of \( B \) (liberally mix \( (x, y) \) and \( (r, \Theta) \) coordinates!)
Calculate and simplify the matrix products \( AB \) and \( BA \).