1.) Find all critical points of the function \( f(x, y) = x^3 - 3x^2 + 9x + y^3 - 3y + 7 \), and determine which of these are local maxima, local minima, saddle points, and which are neither of these.

2.) A ball thrown from ground level with initial velocity \( v \) at an angle \( \alpha \) (with the horizontal) hits the ground again at the horizontal distance

\[
s = \frac{v^2 \sin(2\alpha)}{g}
\]

(where \( g = -9.81 \frac{m}{s^2} \)).

a.) Calculate all first and second order partial derivatives of \( s \).
Is \( \frac{\partial s}{\partial \alpha} \) always positive? What is the practical meaning of this?
Is \( \frac{\partial^2 s}{\partial \alpha^2} \) always positive? What is the practical meaning of this?
b.) Find the maximum of \( s \) if \( 0 \leq \alpha \leq \frac{\pi}{2} \) and \( 0 \leq v \leq 20 \) (meters per second). (Use \( g \approx 10 \) for computational convenience).

3.) Consider the picture given on the right.

a.) Find formulas for the height of the point \( D \), the slope of the secant line through \( A \) and \( D \), and the tangent line (in the \( y \)-direction) through the point \( D \).

b.) Let \( dx = [AB] \) and \( dy = [AE] \), and \( \vec{a} \) be the unit vector in the direction of \( \vec{a} = (\Delta x)i + (\Delta y)j \). Mark the following on the picture: \( dz, \Delta z \), and a line whose slopes is \( (D_2 f)(a, b) \).

c.) If \( z = xy \), \( (a, b) = (5, 7) \), \( dx = 0.08 \), \( dy = 0.06 \) find \( dz \).

4.) Consider the contour diagram given on the left.

a.) Find all saddle points.

b.) Show the directions of the gradient at \( P \).

c.) Which of \( f_x, f_y, f_{xx}, f_{yy} \) are positive at \( P \)?

5.) Consider the given table of values of a function \( w = f(t, x) \).
Estimate the directional derivative \( D_{\vec{u}}(2, 3) \) where \( \vec{u} = (0.6, 0.8) \)
(Hint: First estimate the partial derivatives \( f_t(2, 3) \) and \( f_x(2, 3) \)).

**For bonus credit:** Suppose that this function is a solution of the partial differential equation \( w_t = w_{xx} \). Use this together with the information in the table to estimate \( f(2.2, 3) \).

<table>
<thead>
<tr>
<th>t/x</th>
<th>2.8</th>
<th>2.9</th>
<th>3.0</th>
<th>3.1</th>
<th>3.2</th>
</tr>
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<tbody>
<tr>
<td>1.9</td>
<td>0.324</td>
<td>0.354</td>
<td>0.380</td>
<td>0.402</td>
<td>0.421</td>
</tr>
<tr>
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<td>0.364</td>
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<tr>
<td>2.1</td>
<td>0.265</td>
<td>0.290</td>
<td>0.311</td>
<td>0.330</td>
<td>0.345</td>
</tr>
</tbody>
</table>

6.) Find the line \( y = a + bx \) that minimizes the least squares distance \( z = \sum_{i=1}^{3} (y_i - (ax_i + b))^2 \) from the points \((-1, 2), (0, 3), \) and \((1, 2)\).