Instructions: Justify all work, explain your reasoning in detail, & make sure everything fits neatly together.

1.a) Find formulas for the pictured vector fields, from the list given below:

\[ \pm \hat{i}, \pm \hat{j}, \pm \hat{k}, \pm \hat{i} \pm \hat{j}, \pm \hat{i} \pm \hat{k}, \pm \hat{j} \pm \hat{k}, \pm (\hat{i} + \hat{j}), \pm \frac{\hat{i} + \hat{j}}{\sqrt{x^2 + y^2}}, \pm \frac{\hat{j} + \hat{k}}{\sqrt{x^2 + y^2}}, \pm (\hat{i} + \hat{x} \hat{j}), \pm \frac{\hat{i} + \hat{x} \hat{j}}{\sqrt{x^2 + y^2}}, \pm \frac{\hat{j} + \hat{x} \hat{j}}{\sqrt{x^2 + y^2}}. \]

b.) At least one of the pictured vector fields is the gradient field of a function. Find out which, and explain. Sketch typical contours. (For extra credit: Find a formula for the function.)

c.) At least one pictured vector field cannot be a gradient field. Find out which, explain.

d.) Which fields look irrotational to you? Which appear divergence-free? Give pictorial arguments, and check with the formulas found in a.)

2.) Consider the parametrized curve \( \vec{R}(t) = \hat{i} \cos t^2 + \hat{j} \sin t^2 \) for \( t \geq 0 \).

a.) Describe the curve in words.

b.) Find the velocity \( \vec{v}(t) \) and the acceleration \( \vec{a}(t) \).

c.) Find the speed \( |\vec{v}(t)| \) and sketch its graph.

D.) Using parts a.) - c.), describe the general features of the parallel \( a_x(t) \) and perpendicular \( a_\perp(t) \) components of the acceleration? (E.g. zero, positive, negative, constant, increasing or decreasing . . . ) (Alternatively find formulas and sketch their graphs.)

3.a) Calculate the circulation \( \oint_C \vec{F} \cdot d\vec{R} \) of \( \vec{F}(x, y) = \hat{i} + \hat{j} \) around the triangle \( T \) with corners \( A(0, 0), B(2, 2) \) and \( C(1, 3) \).

b.) What are the curl and the divergence of the vector field \( \vec{G}(x, y, z) = \hat{i} + \hat{j} \)?

c.) Use part b.) and Stokes'/divergence theorem to get a shortcut for part a.) (Aside: What is a good way to find the area of the triangle?) (If you used the theorems already in part a.) demonstrate here that you also can set up and evaluate line integrals directly.)

4.) TAKE HOME PROBLEM. DUE WEDNESDAY. If not working alone, properly acknowledge any collaboration.

Describe the vector field \( \vec{F}(x, y, z) = \frac{1}{(x^2 + y^2 + z^2)^{3/2}} (x\hat{i} + y\hat{j} + z\hat{k}) \) in words. Can you think of a possible practical occurrence?

b.) Calculate the partial derivative \( \frac{\partial}{\partial z} (x^2 + y^2 + z^2)^{3/2} \) and find the divergence \( \text{div} \vec{F}(x, y, z) \).

c.) Evaluate the flux integral \( \iint_S \vec{F} \cdot d\vec{A} \) of \( \vec{F} \) over the sphere with radius \( a \) centered at the origin.

d.) How does the answer of c.) depend on the radius \( a \)? What does this have to do with b.).

Can you think of any practical implications/interpretations?