MAT 272 Calculus and Analytic Geometry 3 12/8/93

Preparations for final exam

Office hours: Thursday 1:00 to 2:00 and Friday 2:00 to 3:00.

Today 2:00 - 3:00

If you have kept neat lab-books, turn them in with the final. (Pick them up after a few days, or next semester.)

The final exam is comprehensive: all math that you have learnt in calculus 1-3 and prerequisites (e.g. watch for the product rule). Carefully think about question 1 below. About half of the test will focus on vector calculus (redo test 4, plus homework problems not covered there). The other half reviews other calc III topics (e.g. saddle points etc., constrained optimization, differentials and chain rule, setting up and evaluating double and triple integrals using suitable coordinates. Don’t forget to go back and forth between formulas, tables, graphs and contour diagrams. Be able to estimate derivatives and integrals from tables and contour diagrams, and calculate them from formulas where feasible.

Grading policy: The final will be worth a total of 200 points plus the number of points you missed on the last test. (E.g. if you had 60 on test 4, get 170 out of 200, then you will earn another \( \frac{170}{200}(100 - 60) = 34 \) for a total of 234 points.

1.) (One of the following.) What is a derivative, what is an integral, and what is the fundamental theorem? (E.g. algebraic definition, geometry, different kinds of, practical meaning and applications, examples, ...). Don’t write too much, and in such a way that someone who has not had calculus can understand!

2.) Given a contour diagram (e.g. rainfall in South America, or fox-density), estimate the partial derivatives at a point, write down an approximate equation of the tangent plane, and estimate the integral over a given region (e.g. South America). Give a practical interpretation of the integral and if meaningful of the derivatives.

3.i) Find all critical points of a cubic function in two variables, e.g. \( f(x, y) = x^3 - 6x - y^2 + y^2 + y \), and determine which are minima, maxima, and which are saddles.

(ii) Still open for credit: Repeat with \( f(x, y) = \int x^2 + t \ln t \ dt \).

(iii) A simple constrained optimization problem (E.g. minimize \( f(x, y) = x^2 + 4y^2 \) subject to the constraint \( 2x + 3y = 5 \). Explain why sometimes Lagrange multipliers are superior to calculus I techniques.

4.) Find the moment of inertia \( J \) of a half sphere (with constant density) with radius \( a \) about its symmetry axis (compare 15.4/19.20). The moment of inertia equals the volume integral \( \iiint_V \rho f^2(x, y, z) \, dV \) where \( \rho \) is the mass density, and \( f \) is the distance of the point \( (x, y, z) \) from the axis of rotation.

Use coordinates so that the half sphere is centered at the origin, and lies above the xy-plane. The z-axis is the axis of rotation, and \( f(x, y, z) = r \), in cylindrical coordinates. Set up the integral in suitable coordinates, and evaluate it. Using that the mass equals \( m = \rho V = \frac{3}{2} \pi a^3 \), the final answer is \( J = \frac{3}{5} ma^2 \).

5.) Chain rule and differentials: Compare quiz 6. Also: If \( x = r \cos \Theta, y = r \sin \Theta \), and we measure \( r = 10, \Theta = \frac{\pi}{6} \) with errors of at most 1% and 2%, what are \( x, y \) and how large may the errors be? (Hints: Translate the relative errors into absolute errors \( \delta r \) and \( \delta \Theta \), then use differentials to relate \( dx \) and \( d\Theta \) to \( dr \) and \( d\Theta \).) What is a general formula relating these differentials?

6.) Parametrized curves: speed, acceleration etc. Compare the word you spelled (graph of speed etc), problem 2 from test 4, and the problem from the preparations for test 4. Watch out for the product and chain rules.


8.) Flux and divergence theorem: compare problem 4 of the test. Or try the following (this combines many topics): Find the flux of the vector field \( \mathbf{F}(x, y, z) = (3 + 2x)^2 + y^2 + (y + z)^2 \) through the triangle with corners \( A(3, 0, 0), B(0, 6, 0) \) and \( C(0, 0, 12) \). Rather than evaluating the integral directly, use the divergence theorem by considering the four sides of the pyramid with corners \( A, B, C \), and \( D(0, 0, 0) \). The flux through all four sides equals the integral of the divergence over the interior. But the volume integral, and the three other area integrals are simple.

1 page handwritten notes o.k. - otherwise closed book, closed notebook.