1.a.) (adapted from Strang, Calculus) Consider the vectors \( \vec{v}_1, \ldots, \vec{v}_{12} \) from the center of a clock to the hours. By removing one, or several of the vectors make the length of sum of the remaining vectors as large as possible.

b.) Let \( \vec{u}_1, \ldots, \vec{u}_{12} \) be the vectors that start at 6 o'clock and point towards the hours 1 o'clock to 12 o'clock. What is the sum \( \vec{u}_2 + \vec{u}_4 + \vec{u}_6 + \vec{u}_8 + \vec{u}_{10} + \vec{u}_{12} \)?

2.a.) What is the geometric relation between \( \vec{v} \) and \( \vec{w} \) if \( \vec{v} \cdot \vec{w} = ||\vec{v}||||\vec{w}|| \). Justify your answer.

b.) Compute the area of the triangle with vertices \( A(0,0,0) \), \( B(1,1,0) \), \( C(1,2,1) \).

c.) Does the point \( D(3,8,4) \) lie in the same plane as the triangle \( ABC \)? Explain why or why not.

d.) Use vectors and dot-products to prove Pythagoras' theorem:

\[
\text{In a right triangle with sides } a, b, c \text{ (with } c \text{ the longest side)} \quad c^2 = a^2 + b^2.
\]

(Hint: Let \( \vec{u} \) and \( \vec{v} \) be vectors along the short sides and compute \( (\vec{v} - \vec{u}) \cdot (\vec{v} - \vec{u}) \). EXPLAIN your reasoning!)

3.a.) For the function \( f(x, y) = x^2y^3 \) find the direction of the steepest increase at the point \( (a, b) = (1,1) \).

b.) Use the value of \( f(1,1) = 1 \) and the gradient to obtain a linear approximation for \( 1.05^2 \cdot 0.97^3 \).

4.) Given below are the gradient fields of three functions of two variables each.

a.) Roughly sketch the contour lines of the functions whose gradient fields are depicted.

b.) Match each of these to a possible graph among the choices:

A. A plane,

C. A circular cone opening upward along the z-axis,

E. A circular paraboloid opening upward,

B. An upper half-sphere centered at the origin,

D. A circular cone opening downward along the z-axis,

F. A circular paraboloid opening downward.

5.a.) Suppose that a function \( f(x,y) \) of two variable has continuous partial derivatives and let \( \vec{u} = (0.8\vec{i} + 0.6\vec{j}) \) and \( \vec{v} = (0.6\vec{i} + 0.8\vec{j}) \). If \( D_{\vec{u}}f(a,b) = 2 \) and \( D_{\vec{v}}f(a,b) = 3 \) find \( \frac{\partial f}{\partial x}(a,b) \) and \( \frac{\partial f}{\partial y}(a,b) \).

b.) Bonus question: Suppose that a function \( f(x,y) \) of two variables has continuous partial derivatives. Is it true that at any given \( (a,b) \) all tangent lines (in all directions) to the graph of \( f(x,y) \) at \( (a,b) \) lie in a plane? Justify your answer in as much detail as possible!