Document all your work. Justify all nontrivial steps, in particular, all steps involving integral theorems. ASK FOR HELP if anything is unclear.

1.) Find a numerical value for the line integral \( \int_C f \, ds \) where \( f(x, y) = x + y \) and the curve \( C \) is the upper half of the unit circle (from \((1,0)\) to \((-1,0)\)). Bonus question: Where is the center of mass of the curve \( C \)?

2.) Let the curve \( C \) be the boundary of the triangle with corners \((0,0)\), \((1,0)\), and \((0,1)\) (oriented counter-clockwise), and let \( \vec{F} \) be the vector field \( \vec{F}(x,y) = (x+y)\vec{j} \).
   a.) Directly evaluate the line integral \( \int_C \vec{F} \cdot \vec{N} \, ds \) for the flux of \( \vec{F} \) across \( C \). (\( \vec{N} \) is the outward unit normal to \( C \)).
   b.) Use an integral theorem to convert the line integral into an area integral, and then evaluate the latter.

3.) Let the surface \( S \) be the part of the graph \( z = g(x,y) = 1-x^2-y^2 \) that lies above the \( xy \)-plane.
   a.) Find the surface area of \( S \), i.e. evaluate the surface integral \( \int_S dS \).
   b.) For the vector field \( \vec{F}(x,y,z) = \vec{i} + z^2\vec{k} \) directly evaluate the integral \( \int_S \vec{F} \cdot \vec{N} \, dS \) for the flux of \( \vec{F} \) across \( S \).
   (Let \( \vec{N} \) be the unit normal vector that points upwards.)

4.a.) Explain in detail how one may use the divergence theorem to write the flux integral of 3.b. as a volume integral.
   b.) Evaluate the volume integral of 4.a.

5.a) Calculate the circulation \( \int_C \vec{F} \cdot d\vec{r} \) of the vector field \( \vec{F} = \sin^2 \pi x \vec{i} + 3z \vec{j} + \sqrt{x^2 + 1} \vec{k} \) around the triangle \( C \) with corners \((0,0,1)\), \((1,0,1)\), and \((1,1,0)\) (traversed in this order). REMARK: The vector field \( \vec{F} \) is cooked-up in such a way that direct evaluation should be very unpleasant, yet applying Stokes' theorem gives a very simple integral — don't waste your time! What is \( \nabla \times \vec{F} \)? What is \( \vec{N} \) and what is the area of a triangle?
   b.) Does there exist a potential \( f \) such that \( \vec{F}(x,y,z) = \nabla f(x,y,z) \)? If yes, find such \( f \), if not, explain why not.

For comparison, using Thomas/Finney: (90% of class failed!)