MAT 394 - PS4 Solutions

1.) Suppose that a fair six-sided die is rolled three times and let X_1, X_2 and X_3 be the three numbers obtained, in order of occurrence.

(a) Calculate the probability that $X_1 < X_2 < X_3$.

(b) Calculate the conditional probability that $X_1 = 1$ given that $X_1 < X_2 < X_3$.

Solutions:

(a) Because the values of X_1, X_2, and X_3 are independent, the probability of any particular sequence of values is equal to $(1/6)^3 = 1/216$. For example,

$$\mathbb{P}(X_1 = 1, X_2 = 2, X_3 = 3) = \mathbb{P}(X_1 = 1)\mathbb{P}(X_2 = 2)\mathbb{P}(X_3 = 3) = \frac{1}{216}.$$

Thus, if we let E denote the event that $X_1 < X_2 < X_3$, then $\mathbb{P}(E)$ can be calculated by first counting the number of ways in which E can occur and then dividing by 216. However, for each sequence of three distinct integers between 1 and 6, there is exactly one way in these can occur in increasing order. Thus the number of elements in E is equal to the number of ways of choosing three distinct numbers from the set $\{1, 2, 3, 4, 5, 6\}$, which is $\binom{6}{3} = 20$. Of course, this number could also be calculated by explicitly listing these possibilities, but combinatorics leads to a quicker solution. Consequently,

$$\mathbb{P}(E) = \frac{20}{216} = \frac{5}{54}.$$

(b) We first calculate the probability of the event $\{X_1 = 1\} \cap E = \{X_1 = 1 < X_2 < X_3\}$. Once we fix $X_1 = 1$, there are $\binom{5}{2} = 10$ possible values for X_2 and X_3 that will satisfy this condition and so

$$\mathbb{P}\left(\{X_1 = 1\} \cap E\right) = \frac{10}{216}.$$

Then,

$$\mathbb{P}(X_1 = 1|E) = \frac{\mathbb{P}\left(\{X_1 = 1\} \cap E\right)}{\mathbb{P}(E)} = \frac{10/216}{20/216} = \frac{1}{2}.$$

2.) Sums of independent random variables.

(a) Suppose that X and Y are independent, integer-valued random variables and let $Z = X + Y$. Explain why the following identity is true:

$$\mathbb{P}(Z = n) = \sum_{k=-\infty}^{\infty} \mathbb{P}(X = k)\mathbb{P}(Y = n - k).$$

(b) Use the result from part (a) to show that if $X \sim \text{Binomial}(n, p)$ and $Y \sim \text{Binomial}(m, p)$, then $Z = X + Y \sim \text{Binomial}(n + m, p)$. Explain why this result is *intuitively* true without doing any calculations. *Hint:* Interpret the binomial distribution in terms of independent trials.

Solutions:

(a) First observe that the event $\{Z = n\}$ is equal to the countable disjoint union of the events $\{X = k, Y = n - k\}$, where k ranges over the integers:

$$\{Z = n\} = \bigcup_{k=-\infty}^{\infty} \{X = k, Y = n - k\}.$$

Then, since X and Y are independent, it follows that

$$\mathbb{P}(X = k, Y = n - k) = \mathbb{P}(X = k)\mathbb{P}(Y = n - k).$$

Lastly, by combining these two results and using the additivity of probabilities, we arrive at the desired identity:

$$\begin{aligned} \mathbb{P}(Z = n) &= \mathbb{P}\left(\bigcup_{k=-\infty}^{\infty} \{X = k, Y = n - k\} \right) \\ &= \sum_{k=-\infty}^{\infty} \mathbb{P}(X = k, Y = n - k) \\ &= \sum_{k=-\infty}^{\infty} \mathbb{P}(X = k)\mathbb{P}(Y = n - k). \end{aligned}$$

(b) Although this result can be derived directly from (a), we can also reason as follows. Since X and Y are binomial random variable, we can interpret X and Y as the numbers of successes obtained when we perform a series of first n and then m independent trials, where each trial has probability p of being successful. Furthermore, since X and Y are independent, we can interpret $X + Y$ as the total number of successes when we perform a series of $n + m$ trials, each of which has success probability p. However, this interpretation makes it clear that $X + Y$ is binomially distributed with parameters $n + m$ and p.

3.) Use the definition of the mean and variance to show that if X is Poisson-distributed with parameter λ, then $\mathbb{E}[X] = \mathrm{Var}(X) = \lambda$.

Solution: Recall that the probability mass function of X is

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}.$$

Therefore, by definition, the expected value of X is

$$\begin{aligned}
\mathbb{E}[X] &= \sum_{k=0}^{\infty} p(k)\cdot k \\
&= e^{-\lambda}\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}\cdot k \\
&= e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k-1)!} \\
&= \lambda e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^{k-1}}{(k-1)!} \\
&= \lambda e^{-\lambda}\sum_{k=0}^{\infty}\frac{\lambda^k}{k!} \\
&= \lambda e^{-\lambda}e^{\lambda} = \lambda,
\end{aligned}$$

where we have used the Taylor series expansion of the exponential function:

$$e^{\lambda} = \sum_{k=0}^{\infty}\frac{\lambda^k}{k!}.$$

Similarly,

$$\begin{aligned}
\mathbb{E}[X^2] &= \sum_{k=0}^{\infty} p(k)\cdot k^2 \\
&= e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k-1)!}\cdot(k-1+1) \\
&= e^{-\lambda}\sum_{k=2}^{\infty}\frac{\lambda^k}{(k-2)!} + e^{-\lambda}\sum_{k=1}^{\infty}\frac{\lambda^k}{(k-1)!} \\
&= \lambda^2 + \lambda.
\end{aligned}$$

Consequently, the variance of X is

$$\mathrm{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \lambda.$$

4.) Show that if $X = \sigma Z + \mu$, where Z is a standard normal random variable and μ and σ are constants with $\sigma > 0$, then X is a normal random variable with mean μ and variance σ^2. *Hint:* Use a change-of-variables to show that X has the "correct" cumulative distribution function (which you will need to express as an integral).

Solution: Let's start by calculating the cumulative distribution function of X:

$$\begin{aligned}
\mathbb{P}(X < x) &= \mathbb{P}(\sigma Z + \mu < x) \\
&= \mathbb{P}\left(Z < \frac{x-\mu}{\sigma}\right) \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(x-\mu)/\sigma} e^{-t^2/2} dt \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-(y-\mu)^2/2\sigma^2} \frac{1}{\sigma} dy \\
&= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-(y-\mu)^2/2\sigma^2} dy,
\end{aligned}$$

where we used the change of variables $t = (y-\mu)/\sigma$ and $dt = dy/\sigma$ in passing from the third to the fourth line. However, since

$$p(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/2\sigma^2}$$

is the probability density function of a normal random variable with mean μ and variance σ^2, it follows that this is the distribution of X itself. (Recall that the probability density function uniquely determines the distribution of a random variable.)

5.) Suppose that a couple, Michael and Nancy, have two children, and that both Michael's brother and Nancy's father's brother's child (i.e., her first cousin) have cystic fibrosis. If (i) both of these cases are caused by the CFTRΔ508 mutation; (ii) the frequencies of the wild type (WW), carrier (WD) and disease genotypes (DD) in this population are 0.9025, 0.095 and 0.0025, respectively; (iii) mating is random with respect to this mutation, i.e., the genotypes of any two individuals forming a couple are independent of one another; and (iv) neither Michael nor Nancy nor any of their parents or aunts or uncles have cystic fibrosis,

(a) what is the probability that both of Michael's parents are carriers of the disease allele?

(b) what is the probability that Nancy's mother is a carrier of the disease allele? *Hint:* What is the probability that an individual chosen at random is a carrier given that they do not have cystic fibrosis?

(c) what is the probability that Nancy's father is a carrier of the disease allele? *Hint:* Consider all of the possible genotypes of Nancy's father's parents.

(d) what is the probability that Nancy and Michael's first child will have cystic fibrosis?

(e) what is the probability that their second child will have cystic fibrosis if their first child does not?

(f) what is the probability that their second child will have cystic fibrosis if their first child does have this condition?

Hint: Start by drawing a pedigree showing all of the individuals that are relevant to the problem. Also, denote the wild type and disease alleles by the letters W and D, respectively.

Solutions:

(a) Since Michael's brother is affected and both of Michael's parents are known to not be affected, it follows that both parents must be carriers of the CF allele, i.e., the probability is equal to 1.

(b) Since nothing is known about Nancy's mother's relatives (apart from Nancy herself), but we do know that Nancy's mother is not affected, it follows that the probability that Nancy's mother is a carrier is equal to the conditional probability of sampling a carrier from the population given that that individual is not affected, i.e.,

$$\mathbb{P}(\text{Nm} = \text{WD} | \text{Nm} \neq \text{DD}) = \frac{0.095}{1 - 0.0025} \approx 0.09524.$$

In this case, since CF cases are very rare, conditioning on not having CF barely changes the probability of being a carrier. Notice, however, that we have not yet used the additional information that Nancy herself is unaffected, although this will be used below.

(c) To calculate the probability that Nancy's father is a carrier, we need to consider the possible genotypes of his parents (individuals Nff and Nfm in the pedigree; cf. Fig. 1). Since we know that Nancy's cousin (Nc) has CF, it follows that Nancy's aunt and uncle (Na, Nu), both of whom are known to be unaffected, must be carriers. However, only the uncle's status is relevant here since the random mating assumption allows us to assume that Nancy's aunt is unrelated to either her uncle or the uncle's parents. Then, with the help of

Bayes' formula (and ignoring, temporarily, the fact that Nu is known to be unaffected; see below), we have

$$\begin{aligned}
\mathbb{P}(\text{Nff = WW, Nfm = WW}|\text{Nu = WD}) &= 0\\
\mathbb{P}(\text{Nff = WD, Nfm = WW}|\text{Nu = WD}) &= \mathbb{P}(\text{Nff = WD, Nfm = WW})\frac{\mathbb{P}(\text{Nu = WD}|\text{Nff = WD, Nfm = WW})}{\mathbb{P}(\text{Nu = WD})}\\
&= \mathbb{P}(\text{Nff = WD})\mathbb{P}(\text{Nfm = WW})\frac{1/2}{\mathbb{P}(\text{Nu = WD})}\\
&= 0.9025 \times \frac{1}{2} = 0.45125\\
\mathbb{P}(\text{Nff = WW, Nfm = WD}|\text{Nu = WD}) &= 0.45125\\
\mathbb{P}(\text{Nff = WD, Nfm = WD}|\text{Nu = WD}) &= \mathbb{P}(\text{Nff = WD, Nfm = WD})\frac{\mathbb{P}(\text{Nu = WD}|\text{Nff = WD, Nfm = WD})}{\mathbb{P}(\text{Nu = WD})}\\
&= \mathbb{P}(\text{Nff = WD})\mathbb{P}(\text{Nfm = WD})\frac{1/2}{\mathbb{P}(\text{Nu = WD})}\\
&= 0.095 \times \frac{1}{2} = 0.0475\\
\mathbb{P}(\text{Nff = DD, Nfm = WW}|\text{Nu = WD}) &= \mathbb{P}(\text{Nff = DD, Nfm = WW})\frac{\mathbb{P}(\text{Nu = WD}|\text{Nff = DD, Nfm = WW})}{\mathbb{P}(\text{Nu = WD})}\\
&= \mathbb{P}(\text{Nff = DD})\mathbb{P}(\text{Nfm = WW})\frac{1}{\mathbb{P}(\text{Nu = WD})}\\
&= 0.0025 \times 0.9025/0.095 = 0.02375\\
\mathbb{P}(\text{Nff = WW, Nfm = DD}|\text{Nu = WD}) &= 0.02375\\
\mathbb{P}(\text{Nff = DD, Nfm = WD}|\text{Nu = WD}) &= \mathbb{P}(\text{Nff = DD, Nfm = WD})\frac{\mathbb{P}(\text{Nu = WD}|\text{Nff = DD, Nfm = WD})}{\mathbb{P}(\text{Nu = WD})}\\
&= \mathbb{P}(\text{Nff = DD})\mathbb{P}(\text{Nfm = WD})\frac{1/2}{\mathbb{P}(\text{Nu = WD})}\\
&= 0.0025 \times \frac{1}{2} = 0.00125\\
\mathbb{P}(\text{Nff = WD, Nfm = DD}|\text{Nu = WD}) &= 0.00125\\
\mathbb{P}(\text{Nff = DD, Nfm = DD}|\text{Nu = WD}) &= 0,
\end{aligned}$$

and a quick calculation verifies that these sum to 1, as they must.

Two points deserve mention here. First, notice that because we have assumed that mating is random in this population, the genotypes of Nff and Nfm are independent so long as we do not take into account the genotypes of any of their children. This is why, for example, the following identity is true

$$\mathbb{P}(\text{Nff = WD, Nfm = WW}) = \mathbb{P}(\text{Nff = WD})\mathbb{P}(\text{Nfm = WW}) = 0.095 \times 0.9025.$$

Of course, independence is lost as soon as we condition on the genotypes of their offspring, since these depend on the genotypes of both parents. Secondly, we have also used the fact that the unconditional probability that Nu is a carrier is just $\mathbb{P}(\text{Nu = WD}) = 0.095$, which is correct here because have not made use of the additional information that Nu is unaffected. We could also have calculated the probabilities of the parental genotypes by conditioning on the event Nu $\neq$ DD (and we would have arrived at the same numbers shown above), but notice that this information is redundant once we condition on the event that Nu is a carrier

and thus the approach taken here is more straightforward. The moral is that you should always eliminate redundant information, but you need to do so consistently.

With these probabilities in hand, we can now use the law of total probability to calculate the probability that Nancy's father is a carrier of the CF allele conditional on him not being affected. Excluding the two cases with zero probability, we have

$$\begin{aligned}
\mathbb{P}(\text{Nf} = \text{WD}|\text{Nu} = \text{WD}) &= \mathbb{P}(\text{Nf} = \text{WD}|\text{Nff} = \text{WD}, \text{Nfm} = \text{WW}) \times \mathbb{P}(\text{Nff} = \text{WD}, \text{Nfm} = \text{WW}|\text{Nu} = \text{WD}) + \\
&\quad \mathbb{P}(\text{Nf} = \text{WD}|\text{Nff} = \text{WW}, \text{Nfm} = \text{WD}) \times \mathbb{P}(\text{Nff} = \text{WW}, \text{Nfm} = \text{WD}|\text{Nu} = \text{WD}) + \\
&\quad \mathbb{P}(\text{Nf} = \text{WD}|\text{Nff} = \text{WD}, \text{Nfm} = \text{WD}) \times \mathbb{P}(\text{Nff} = \text{WD}, \text{Nfm} = \text{WD}|\text{Nu} = \text{WD}) + \\
&\quad \mathbb{P}(\text{Nf} = \text{WD}|\text{Nff} = \text{DD}, \text{Nfm} = \text{WW}) \times \mathbb{P}(\text{Nff} = \text{DD}, \text{Nfm} = \text{WW}|\text{Nu} = \text{WD}) + \\
&\quad \mathbb{P}(\text{Nf} = \text{WD}|\text{Nff} = \text{WW}, \text{Nfm} = \text{DD}) \times \mathbb{P}(\text{Nff} = \text{WW}, \text{Nfm} = \text{DD}|\text{Nu} = \text{WD}) + \\
&\quad \mathbb{P}(\text{Nf} = \text{WD}|\text{Nff} = \text{DD}, \text{Nfm} = \text{WD}) \times \mathbb{P}(\text{Nff} = \text{DD}, \text{Nfm} = \text{WD}|\text{Nu} = \text{WD}) + \\
&\quad \mathbb{P}(\text{Nf} = \text{WD}|\text{Nff} = \text{WD}, \text{Nfm} = \text{DD}) \times \mathbb{P}(\text{Nff} = \text{WD}, \text{Nfm} = \text{DD}|\text{Nu} = \text{WD}) \\
&= \frac{1}{2} \times 0.45125 + \frac{1}{2} \times 0.45125 + \frac{1}{2} \times 0.0475 + \frac{1}{2} \times 0.02375 + \frac{1}{2} \times 0.00125 + \frac{1}{2} \times 0.00125 \\
&= 0.488125.
\end{aligned}$$

However, we are not quite done, since we still need to condition on the information that Nancy's father is not affected. To do this, we first calculate the conditional probability that he is affected given the status of Nc. This can be done as above, but is even easier:

$$\mathbb{P}(\text{Nf} = \text{DD}|\text{Nu} = \text{WD}) = 0.013125.$$

Then, since $\{\text{Nf} = \text{WD}\} \cap \{\text{Nf} \neq \text{DD}\} = \{\text{Nf} = \text{WD}\}$, it follows that the probability that we are interested in is

$$\begin{aligned}
\mathbb{P}(\text{Nf} = \text{WD}|\text{Nf} \neq \text{DD}, \text{Nu} = \text{WD}) &= \frac{\mathbb{P}(\text{Nf} = \text{WD}|\text{Nu} = \text{WD})}{\mathbb{P}(\text{Nf} \neq \text{DD}|\text{Nu} = \text{WD})} \\
&= \frac{\mathbb{P}(\text{Nf} = \text{WD}|\text{Nu} = \text{WD})}{1 - \mathbb{P}(\text{Nf} = \text{DD}|\text{Nu} = \text{WD})} \\
&= \frac{0.488125}{1 - 0.013125} \approx 0.49462.
\end{aligned}$$

That this probability is almost equal to 1/2 reflects the fact that, on average, full sibs (e.g., Nf and Nu) share half of their genes in common. That it is not exactly equal to 1/2 is due to the additional information that we have concerning the allele frequencies and the fact that Nf is not affected.

(d) To solve this part of the problem, we first need to calculate the probability that Nancy and Michael are both carriers. Since Michael's parents are both carriers and Michael is known to not be affected, it follows that the probability that he is a carrier is equal to

$$\mathbb{P}(\text{Michael} = \text{WD}|\text{Mf} = \text{Mm} = \text{WD}, \text{Michael} \neq \text{DD}) = \frac{1/2}{1 - 1/4} = 2/3.$$

To calculate the probability that Nancy is a carrier, we again need to use the law of total probability. We will first ignore the additional information that she is not affected. Since the genotypes of her parents are

independent of one another, it follows that

$$\begin{aligned}
&\mathbb{P}(\text{Nancy} = \text{WD}|\text{Nu} = \text{WD}) = \\
&\quad \mathbb{P}(\text{Nancy} = \text{WD}|\text{Nf} = \text{WW}, \text{Nm} = \text{WW})\mathbb{P}(\text{Nf} = \text{WW}|\text{Nu} = \text{WD})\mathbb{P}(\text{Nm} = \text{WW}) + \\
&\quad \mathbb{P}(\text{Nancy} = \text{WD}|\text{Nf} = \text{WD}, \text{Nm} = \text{WW})\mathbb{P}(\text{Nf} = \text{WD}|\text{Nu} = \text{WD})\mathbb{P}(\text{Nm} = \text{WW}) + \\
&\quad \mathbb{P}(\text{Nancy} = \text{WD}|\text{Nf} = \text{WW}, \text{Nm} = \text{WD})\mathbb{P}(\text{Nf} = \text{WW}|\text{Nu} = \text{WD})\mathbb{P}(\text{Nm} = \text{WD}) + \\
&\quad \mathbb{P}(\text{Nancy} = \text{WD}|\text{Nf} = \text{WD}, \text{Nm} = \text{WD})\mathbb{P}(\text{Nf} = \text{WD}|\text{Nu} = \text{WD})\mathbb{P}(\text{Nm} = \text{WD}) \\
&= 0 + \frac{1}{2} \times \left[0.488125 \times (1 - 0.09524) + (1 - 0.488125) \times 0.09524 + 0.488125 \times 0.09524\right] \\
&\approx 0.26844,
\end{aligned}$$

with the zero in the next to last line coming from the fact that Nancy cannot be a carrier if neither of her parents is a carrier. A similar calculation shows that

$$\mathbb{P}(\text{Nancy} = \text{DD}|\text{Nu} = \text{WD}) \approx 0.01162$$

and thus the probability that Nancy is a carrier given that she is not affected is equal to

$$\mathbb{P}(\text{Nancy} = \text{WD}|\text{Nu} = \text{WD}, \text{Nancy} \neq \text{DD}) = \frac{0.26844}{1 - 0.01162} \approx 0.2716.$$

It is not surprising that Nancy is less likely to be a carrier than Michael, since we know that both of Michael's parents are carriers while we only know that Nancy's uncle is a carrier.

Finally, for Nancy and Michael's first child to be affected, both parents must be carriers and thus

$$\begin{aligned}
\mathbb{P}(\text{c1} = \text{DD}) &= \mathbb{P}(\text{c1} = \text{DD}|\text{Nancy} = \text{Michael} = \text{WD})\mathbb{P}(\text{Nancy} = \text{Michael} = \text{WD}) \\
&= \frac{1}{4} \times \frac{2}{3} \times 0.2716 \approx 0.04527,
\end{aligned}$$

where we have again used the random mating assumption in the second line. Although this probability is fairly small, it is still approximately 17.9 times greater than that of a random individual, reflecting the increased risk due to presence of CF cases in both parents' families. Calculations like these are carried out by genetic counsellors and the results can be used to help inform decisions about marriage as well as genetic screening and pre-natal testing.

(e) We begin by using Bayes' formula to calculate the conditional probabilities of the parents' genotypes

given that the first child does not have cystic fibrosis:

$$\begin{aligned}
&\mathbb{P}(\text{Michael} = \text{WW, Nancy} = \text{WW}|\text{c1} \neq \text{DD}) \\
&\quad = \mathbb{P}(\text{Michael} = \text{WW, Nancy} = \text{WW})\frac{\mathbb{P}(\text{c1} \neq \text{DD}|\text{Michael} = \text{WW, Nancy} = \text{WW})}{\mathbb{P}(\text{c1} \neq \text{DD})} \\
&\quad \approx \frac{1}{3} \times 0.7284 \times \frac{1}{1 - 0.04527} \approx 0.25431 \\
&\mathbb{P}(\text{Michael} = \text{WD, Nancy} = \text{WW}|\text{c1} \neq \text{DD}) \\
&\quad = \mathbb{P}(\text{Michael} = \text{WD, Nancy} = \text{WW})\frac{\mathbb{P}(\text{c1} \neq \text{DD}|\text{Michael} = \text{WD, Nancy} = \text{WW})}{\mathbb{P}(\text{c1} \neq \text{DD})} \\
&\quad \approx \frac{2}{3} \times 0.7284 \times \frac{1}{1 - 0.04527} \approx 0.50863 \\
&\mathbb{P}(\text{Michael} = \text{WW, Nancy} = \text{WD}|\text{c1} \neq \text{DD}) \\
&\quad = \mathbb{P}(\text{Michael} = \text{WW, Nancy} = \text{WD})\frac{\mathbb{P}(\text{c1} \neq \text{DD}|\text{Michael} = \text{WW, Nancy} = \text{WD})}{\mathbb{P}(\text{c1} \neq \text{DD})} \\
&\quad \approx \frac{1}{3} \times 0.2716 \times \frac{1}{1 - 0.04527} \approx 0.09483 \\
&\mathbb{P}(\text{Michael} = \text{WD, Nancy} = \text{WD}|\text{c1} \neq \text{DD}) \\
&\quad = \mathbb{P}(\text{Michael} = \text{WD, Nancy} = \text{WD})\frac{\mathbb{P}(\text{c1} \neq \text{DD}|\text{Michael} = \text{WD, Nancy} = \text{WD})}{\mathbb{P}(\text{c1} \neq \text{DD})} \\
&\quad \approx \frac{2}{3} \times 0.2716 \times \frac{3/4}{1 - 0.04527} \approx 0.14224.
\end{aligned}$$

As a check on our calculations, we note that these four probabilities sum to one. We can then use the law of total probability to calculate the conditional probability that the second child will be affected given that the first one is not. Since the second child can be affected only if both parents are carriers, this calculation is simple:

$$\begin{aligned}
\mathbb{P}(\text{c2 is DD}|\text{c1} \neq \text{DD}) &= \mathbb{P}(\text{c2 is DD}|\text{Michael} = \text{Nancy} = \text{WD})\mathbb{P}(\text{Michael} = \text{Nancy} = \text{WD}|\text{c1} \neq \text{DD}) \\
&\approx \frac{1}{4} \times 0.14224 \approx 0.03556,
\end{aligned}$$

which is slightly smaller than the probability that the first child is affected.

(f) If c1 has cystic fibrosis, then both Michael and Nancy are carriers of the CF allele, in which case their second child will have probability 1/4 of also being affected.

See page 10 for Figure 1.

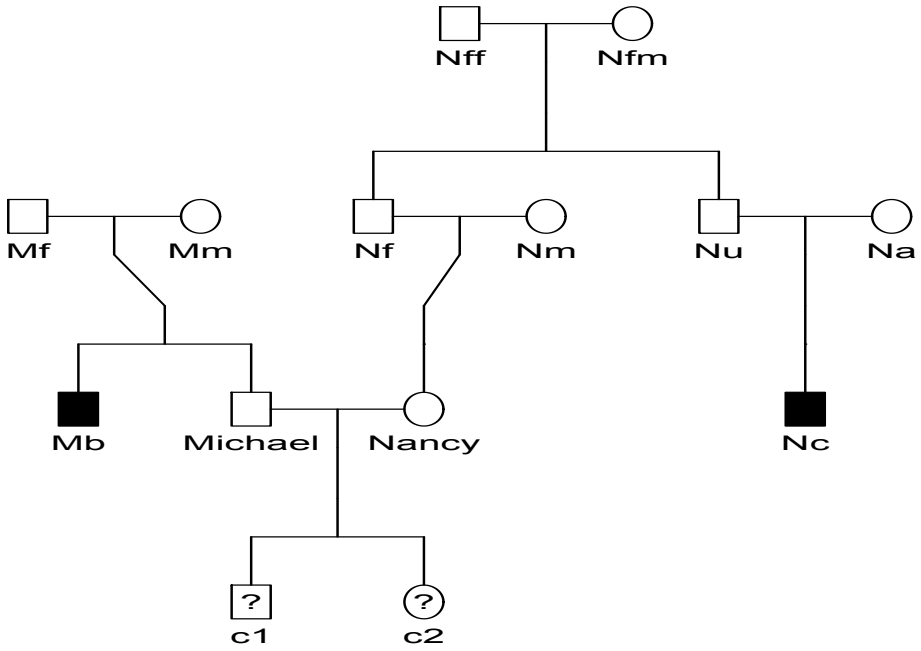

Figure 1: Pedigree for problem 5. Individuals known to have CF are shown in black.