Chapter 0

Basic Maple

0.1 Entering and editing commands

This section describes the basic syntax of Maple commands, which resembles that of conventional programming languages like C, Pascal and Fortran. The basic rules are easy to master, even if you have no previous programming experience.

Maple can manipulate numerical and algebraic expressions. For example, type the expression

\[ 2 + 3; \]

and press the return key. Maple responds with

\[ 5 \]

as expected. Notice:

- *All statements end with a semicolon.* If you hit the return key before you type the semicolon, then Maple does not do anything. In this case, simply type the semicolon on the next line, then hit the return key.

- *Spaces around the plus sign are optional.*

- *You can correct typographical errors by hitting the backspace key,* which erases the previously typed character.

- *The arrow keys can be used to move the cursor.* You can insert new characters simply by typing them, and you can delete characters by depressing the backspace key. Some versions of Maple allow you to edit commands by moving the mouse to position the cursor.
Exercises

1. Enter the command

\[ 2 \cdot 1.3 ; \]

in the following ways:

(a) by eliminating all spaces between the characters;
(b) by inserting one or more spaces before the semicolon;
(c) by typing \(2\cdot1.3\) followed by the return key. Now type a semicolon, followed by the return key.
(d) Use the arrows keys and/or the mouse to change what you have typed to

\[ 2\cdot1.3; \]

and depress the return key.

2. Enter the following command, where the character following the first 2 is a lower case letter "o" instead of a zero:

\[ 2o + 2; \]

How does Maple respond? Edit the command to replace the "o" with a zero.

0.2 Arithmetic operations and precedence

The basic arithmetic operations in Maple are:

<table>
<thead>
<tr>
<th>operation</th>
<th>character</th>
</tr>
</thead>
<tbody>
<tr>
<td>addition</td>
<td>+</td>
</tr>
<tr>
<td>subtraction</td>
<td>-</td>
</tr>
<tr>
<td>multiplication</td>
<td>*</td>
</tr>
<tr>
<td>division</td>
<td>/</td>
</tr>
<tr>
<td>exponentiation</td>
<td>^</td>
</tr>
</tbody>
</table>

*Precedence* refers to the order in which multiple arithmetic operations are performed in a given statement. Algebraic expressions are evaluated from left to right. The following mathematical precedence rules apply:

<table>
<thead>
<tr>
<th>highest</th>
<th>( ) parentheses</th>
</tr>
</thead>
<tbody>
<tr>
<td>next highest</td>
<td>^</td>
</tr>
<tr>
<td>third highest</td>
<td>* /</td>
</tr>
<tr>
<td>lowest</td>
<td>+ -</td>
</tr>
<tr>
<td></td>
<td>exponentiation</td>
</tr>
<tr>
<td></td>
<td>multiplication and division</td>
</tr>
<tr>
<td></td>
<td>addition and subtraction</td>
</tr>
</tbody>
</table>

Example 1. The Maple expression

\[(2 * 3 - 1) ^ 3\]

is evaluated as follows. First, the term in parentheses is evaluated. Because multiplication has higher precedence than subtraction, the multiplication is done first. Therefore, the parenthesized term is equal to 5. Next, the exponentiation is performed. The expression yields 125. □
0.3. EXACT AND FLOATING-POINT ARITHMETIC

Exercises

1. Determine the value of the following expressions before you enter them into Maple. Then check your answers with Maple.
   (a) $3 \times 4 + 5$
   (b) $3 \times 4 \div 5$
   (c) $3 \div 4 \times 5$
   (d) $3 \div 4 \div 5$
   (e) $(1 + 2 \times 3) - 2$
   (f) $2 + 16 \div 2 - 2$
   (g) $9 - 1/2$
   (h) $(3 - 2 + 4 - 2) \div 1/2$
   (i) $4 \div (3/2)$

2. Insert parentheses to make each expression in the following list equal to 8.
   (a) $2 + 2 \times 2$
   (b) $2 \times 2 \times 2$
   (c) $2 \times 2 \div 2$
   (d) $2 \div 2 + 2 \div 2 \times 2$

0.3 Exact and floating-point arithmetic

Consider the following Maple session.

```maple
> 2 / 3;
2
3

> 2.0 / 3;
.6666666666

> evalf(2/3);
.6666666667
```

When each numerical value in an expression is an integer, Maple treats the expression as though it were a fraction. Maple manipulates the expression "exactly" and often uses algebraic identities to simplify it. In contrast, if one or more of the numerical values is a floating-point number (i.e., contains a decimal point), then Maple substitutes a decimal approximation to the expression, which by default is accurate to 10 significant digits. The evalf function returns a floating-point approximation to a given quantity. As the example above shows, the result of evalf may vary slightly from other manipulations with floating-point numbers, depending on how the results are rounded.

Floating point approximations are subject to roundoff errors, because only a finite number of digits are kept in intermediate computations. As a result, floating-point arithmetic sometimes does not satisfy mathematical identities. For this reason, floating-point approximations must be used with care.
Example 1. Consider the following Maple session.

\[
> \sin(100 \times \pi); \\
\text{0}
\]

\[
> \sin(\text{evalf}(100 \times \pi)); \\
.4102067615 \times 10^{-7}
\]

Maple recognizes the expression \(\sin(100 \times \pi)\) as \(\sin 100\pi\). (\(\pi\) is the built-in name for \(\pi\). It is always spelled with an upper-case \(P\) and lower-case \(i\).) Maple repeatedly uses the identity \(\sin(x+2\pi) = \sin x\) to simplify \(\sin 100\pi\) to 0. In contrast, the expression \(\text{evalf}(\sin(100 \times \pi))\) causes Maple to substitute a floating-point approximation for \(100\pi\), whose finite accuracy yields a sine value that is not exactly equal to 0; in this case, the floating-point result is approximately \(0.41 \times 10^{-7}\). □

Exercises

1. Enter each of the following expressions into Maple. What is the mathematically correct answer? Verify that Maple gives the mathematically correct answer whenever the floating-point number is replaced with the corresponding “exact” constant. (Except for \(\text{evalf},\) each of the following Maple functions is equivalent to one that you know from calculus.)

(a) \(\tan(\text{evalf}(\pi))\);
(b) \(\arcsin(-1.0)\);
(c) \(\ln(\exp(1.0))\);

Note the parentheses for the arguments of all Maple functions.

2. What happens if you change the capitalization in the previous exercise? Describe how Maple responds if you type

\[
\text{LN}(\text{EXP}(1.0)));
\]

or

\[
\text{Ln}(\text{Exp}(1.0));
\]

0.4 Variables

The previous discussion shows that Maple can be used as a desk calculator. However, the real power of Maple is its ability to manipulate algebraic expressions, as the following session shows.

\[
> 1 + x + x; \\
1 + 2x
\]

\[
> (1 + 2x)^3; \\
(1 + 2x)^3
\]

\[
> \text{expand}("); \\
1 + 6x + 12x^2 + 8x^3
\]
A variable is a name that is not a constant and not a function. In the above session, x is a variable. Notice that Maple performs some algebraic simplifications automatically. The expand command expands the expression \((1+2x)^3\). This session also illustrates a handy Maple shorthand: the double quotation mark (" ") can be used to refer to the most recently computed result. (In other words, the value of " changes after each command. Here we have used " to refer to the expression \((1+2x)^3\).)

Variables can be assigned specific values, which can be constants, other expressions, or even equations, as the following session illustrates.

\[
\begin{align*}
&> \ x := 3; \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad x := 3 \\
&> \ 1 + 2 \times x; \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 7 \\
&> \ x := 10; \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad x := 10 \\
&> \ 1 + 2 \times x; \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 21 \\
&> \ x := \text{'}x\text{'}; \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad x := x \\
&> \ 1 + 2 \times x; \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 1 + 2x 
\end{align*}
\]

The operator := is used to assign a value to a variable. The assignment \(x := 3\) causes Maple to replace every occurrence of \(x\) with 3 until and unless the value of \(x\) is changed by another assignment statement.

Variables can be unassigned at any time using a command of the form \(x := \text{'}x\text{'}\). (That is, the name of the variable is placed in single quotation marks.) This assignment removes any previous value of \(x\), as illustrated above.

Here are some important points about variables and assignment statements.

- All variables are assigned values with the := operator, as illustrated above.
- Maple variables are not restricted to a single letter. For instance, you could replace every instance of \(x\) with a name like answer. Variables may contain decimal digits, but they must start with a letter. For example, \(x2\) is a valid name for a Maple variable, but \(2x\) is not. Variable names may not contain embedded punctuation like periods or commas, but underscore characters (_) are acceptable.
• In mathematics, adjacent letters usually imply multiplication. (For example, the expression \(xy\) means \(x \times y\).) This convention causes no ambiguity because mathematical unknowns are restricted to a single letter. In Maple, however, you must insert an asterisk (*) between two variables to multiply them. Try the following commands:

\[
x := 3;
y := 5;
x y;
\]

• Maple is case-sensitive; that is, A and a refer to different variables. Most Maple functions are entered in lower case. For instance, sin refers to the sine function, but Sin and SIN do not. (Be careful with the “caps lock” key on your keyboard.)

• It is wise to avoid names like sin, plot, and do for variables, because these names conflict with Maple keywords and built-in functions.

Some variable names represent system constants whose values cannot be overwritten, as the following session illustrates:

\[
> \text{gamma} := 12;
\]

Error, attempting to assign to 'gamma' which is protected

\[
> \text{gamma};
\]

\(\gamma\)

Because \(\text{gamma}\) is a system constant, Maple does not allow you to assign a value to it.

Algebraic expressions can be assigned to variables, as the following Maple session illustrates.

\[
> \text{line} := 1 + 2 \times x;
\]

\(\text{line} := 1 + 2x\)

\[
> 2 \times \text{line};
\]

\(2 + 4x\)

\[
> \text{subs}(x = 3, \text{line});
\]

\(7\)

\[
> \text{line} * \text{line};
\]

\((1 + 2x)^2\)
The first assignment statement causes Maple to substitute \((1 + 2x)\) for every occurrence of the variable line in subsequent expressions.\(^1\)

The \texttt{subs} command can be used to assign temporary values to variables. Notice that the substitution of 3 for \(x\) is indicated using an equals sign \((x = 3)\) instead of the \(:=\) operator. The last command in the sequence above shows that the effect of the \texttt{subs} command is temporary.

Here is another example. Suppose you want to tabulate the values of the polynomial \(z^3 - z + 1\) for various values of \(z\), say for \(z = 0, 1, 2, 3\). The following Maple session shows one way to accomplish this.

\[
\begin{align*}
> & \text{cubic := } z^3 - z + 1; \\
& \text{cubic := } z^3 - z + 1 \\
> & \text{subs(z=0, cubic);} \\
& \quad 1 \\
> & \text{subs(z=1, cubic);} \\
& \quad 1 \\
> & \text{subs(z=2, cubic);} \\
& \quad 7 \\
> & \text{subs(z=3, cubic);} \\
& \quad 25 \\
\end{align*}
\]

The \texttt{subs} command can be used to make temporary assignments to more than one variable in a given expression, as the following session shows.

\[
\begin{align*}
> & \text{expr := } x \ast y; \\
& \text{expr := } x \ast y \\
> & \text{subs(x = 2, y = w + 3, expr);} \\
& \quad 2w + 6 \\
\end{align*}
\]

\(^1\)Be sure that you have unassigned \(x\) if you have made a previous assignment in your Maple session.
Figure 0.1: (a) A plot of $1 + 2x$ for $0 \leq x \leq 3$; (b) a plot of $1 + 2x$ and $x^2$ on the same axes.

Exercises

1. Type in each of the following Maple expressions:
   \[
   xy; \\
   x * y;
   \]
   How does Maple reflect the difference in meaning between the two expressions when it echoes each command?

2. What is the output from the following sequence of Maple commands?
   \[
   p := x^2 \cdot y - 2; \\
   subs(x=2, p); \\
   subs(x=2, y=3, p); \\
   solve(subs(y=1, p), x); \\
   solve(p, x);
   \]

3. Let cubic be a Maple variable corresponding to the polynomial expression $x^3 - x + 1$, as discussed in the text. Describe the output of the Maple command
   \[
   subs(z = y + 1, cubic);
   \]
   where $y$ refers to another unknown. Is this a legal use of subs? If so, what does it produce?

4. Let $x$ be a variable. What is the result of each of the following commands?
   \[
   \text{evalf}(x); \\
   \text{evalf}(1 + 2*x);
   \]

0.5 Plotting functions

We can generate a plot of the function $f(x) = 1 + 2x$ as follows:
   \[
   \text{line} := 1 + 2*x; \\
   \text{plot(line, x = 0 .. 3)};
   \]

Figure 0.1(a) shows the resulting plot.
0.5. **PLOTTING FUNCTIONS**

The first argument to the `plot` command tells Maple what to plot. The second argument gives the range over which the expression is to be plotted. In this example, line refers to the expression \(1 + 2x\), and the syntax \(x = 0 \ldots 3\) specifies that \(x\) should be plotted in the interval \(0 \leq x \leq 3\). Maple displays a graph on the screen that resembles the one shown in Fig 0.1(a). Note that the command

\[
\text{plot}(1 + 2x, x = 0 \ldots 3);
\]

produces the same plot.

Two or more curves can be plotted on the same axes. For instance, let

\[
\text{parabola} := x^2;
\]

The command

\[
\text{plot}([\text{line}, \text{parabola}], x = 0 \ldots 3);
\]

plots \(x^2\) and \(1 + 2x\) on the same axes for \(0 \leq x \leq 3\), as shown in Fig. 0.1(b).

Notice the following:

- Maple uses a different color for each curve on a color monitor.

- Unless you specify otherwise, Maple chooses the limits and increments of the axes. Later we will show you how to override Maple’s choices for the limits on the axes.

- The double dots (\(\ldots\)) separate the lower and upper limits of the horizontal axis. We recommend that you always surround them by spaces. Spaces enhance readability and also prevent misinterpretations, such as when you want \(x\) to run from 0 to .3; the best way to type in such a range is

\[
\text{plot}([\text{line}, \text{parabola}], x = 0 \ldots .3);
\]

**Exercises**

1. Exhibit a sequence of Maple commands to plot \(\sin 2x\) and \(\cos 2x\) on the same set of axes for one full period of each function.

2. Explain why the following sequence of commands does not produce the intended plot.

\[
\begin{align*}
\text{a} & := 1; \\
\text{b} & := 1; \\
\text{plot}(\text{a*sin(b*x)}, x = 0 .. 2*\text{Pi});
\end{align*}
\]

3. Let line be defined as \(1+2x\), as in the previous discussion. Plot line and \(e^x\) (given as `exp(x)` in Maple) on the same set of axes for \(0 \leq x \leq 10\). Explain why the plot looks odd.

4. Explain why the following sequence of commands produces an error message.

\[
\begin{align*}
\text{f1} & := \cos x; \\
\text{f2} & := \sin x; \\
\text{plot}([\text{f1}, \text{f2}], x = 0 .. 2*\text{Pi}); \\
\text{solve}\left(\text{subs}(y = 1, p), x\right); \\
\text{solve}(p, x);
\end{align*}
\]

5. Consider the function

\[
a \sin(bx + c).
\]

(0.1)
0.6  The solve and fsolve commands

One of the principal uses of Maple is to solve equations. Suppose you want to know whether there is a real number \( x \) that satisfies

\[
x = e^{-x}.
\]  

(0.2)

The following session illustrates some useful commands for this purpose.

\[
> \text{eqn} := x = \exp(-x); \\
\text{eqn} := x = e^{-x}
\]

\[
> \text{lhs(eqn)};
\]

\[
x
\]

\[
> \text{rhs(eqn)};
\]

\[
e^{-x}
\]

\[
> \text{plot}([\text{lhs(eqn)}, \text{rhs(eqn)}], x = 0 .. 2);
\]

Here \( \text{eqn} \) is assigned the equation given by (0.2). Notice the different uses of = in the command: the sequence := is the Maple assignment operator, and the second = represents mathematical equality.

There are several ways to solve Eq. (0.2) in Maple. One way to determine whether Eq. (0.2) has a real solution is to plot the left- and right-hand sides. If the curves cross, then the equation has a solution. The plot command in the session above was generated based on the assumption that the solution lies in the interval [0, 2]. The functions \( \text{lhs} \) and \( \text{rhs} \) extract the left-hand side and right-hand side, respectively, of the equation. Figure 0.2 shows the plot. As you can see, the curves cross near \( x = 1/2 \), so Eq. (0.2) has a solution.

The Maple function \( \text{fsolve} \) attempts to find numerical approximations to the solutions of equations. The following continuation of the session above shows how to use \( \text{fsolve} \).

\[
> \text{fsolve(eqn, x = 0 .. 1)};
\]

\[
.5671432904
\]
> fsolve(eqn, x = 1 .. 2);
  
  fsolve\left( x = e^{-x}, x, 1..2 \right)

There is a solution of Eq. (0.2) in the interval [0, 1], and \texttt{fsolve} finds it. Notice that the second argument in \texttt{fsolve} restricts the search to the specified interval. There is no solution of Eq. (0.2) in the interval [1, 2], and Maple indicates its failure to find a solution by echoing the \texttt{fsolve} command.

Maple has a related command, called \texttt{solve}, that attempts to find analytic solutions of a given equation. The \texttt{solve} command attempts to express solutions in terms of radicals, powers, and ratios of integers and constants like \( \pi \) and \( e \). Of course, it is not always possible to express the solutions of a given equation in such terms, so \texttt{solve} is not guaranteed to find an answer. However, \texttt{solve} works well for polynomials and certain trigonometric functions. The following Maple session illustrates the difference between \texttt{solve} and \texttt{fsolve}.

> eqn := sin(x) = cos(x);

\[
eqn := \sin(x) = \cos(x)
\]

> solve(eqn, x);

\[
\frac{1}{4} \pi
\]

> fsolve(eqn, x = 0 .. Pi/2);

\[
.7853981634
\]
Exercises

1. Exhibit a sequence of Maple commands to find a solution of the equation \( e^{-x} = 2x \). Generate a plot and use \texttt{fsolve} to find a numerical approximation to the solution.

2. What happens if you enter the command

\[
\text{plot(eqn, x = 0 .. 2);}
\]

where \texttt{eqn} refers to the equation \( x = e^{-x} \)?

3. Exhibit a sequence of Maple commands to find the roots of the polynomial \( x^5-x^4-5x^3+5x^2+6x-6 \). Find the exact roots, if possible, as well as numerical approximations to each of them. Show how you can avoid typing in the polynomial more than once.

4. Find all solutions of \( \cos x = \sin 3x \) in the interval \( 0 \leq x \leq 2\pi \). (You might find a plot helpful.)

5. How many times does the graph of \( \sin(10x-1) \) cross the \( x \) axis between 0 and 2? Find a numerical approximation to the largest root in this interval. What is the "exact" answer? (Use a trigonometric identity if you cannot get \texttt{solve} to give you the root that you want.)

0.7 Differentiation and differential equations

In this section, we introduce you to some of Maple's capabilities that are needed in the study of differential equations. We begin with a discussion of Maple's differentiation command, called \texttt{diff}. Maple can perform differentiation with respect to any number of variables in a given expression. In this text, we are primarily interested in derivatives with respect to a single variable.

The following Maple session computes the first, second, and tenth derivatives of the polynomial \( 1 + 2x^{11} \).

\[
\begin{align*}
> & \text{expr := 1 + 2 * x^11;} \\
& \text{expr := 1 + 2 x^{11}} \\
> & \text{diff(expr, x);} \\
& 22 x^{10} \\
> & \text{diff(expr, x, x);} \\
& 220 x^9 \\
> & \text{diff(expr, x$2$);} \\
& 220 x^8 \\
> & \text{diff(expr, x$10$);} \\
& 79833600 x
\end{align*}
\]
0.7. DIFFERENTIATION AND DIFFERENTIAL EQUATIONS

The dollar-sign notation is convenient when higher order derivatives are required. Notice that \texttt{diff(expr,x,x)} and \texttt{diff(expr,x$2$)} are equivalent. The last command in the session above computes the tenth derivative.

At times, Maple produces lengthy expressions when it computes derivatives. The \texttt{simplify} command is useful in such situations, as the following session illustrates:

\begin{verbatim}
> p := (x-1) * (x-2) * (x-3);
p := (x-1)(x-2)(x-3)

> diff(p, x);
(x-2)(x-3) + (x-1)(x-3) + (x-1)(x-2)

> simplify(");
3x^2 - 12x + 11
\end{verbatim}

Various combinations of \texttt{simplify} and \texttt{expand} often can be used to find more convenient representations of a given expression.

Differential equations are specified using the \texttt{diff} command. The following Maple session shows the correct way and an incorrect way to enter the exponential growth equation,

\[ x' = kx \quad (0.3) \]

\begin{verbatim}
> good := diff(x(t), t) = k * x(t);
good := \frac{\partial}{\partial t} x(t) = k x(t)

> bad := diff(x, t) = k * x;
bad := 0 = k x
\end{verbatim}

Notice that it is always necessary to specify the dependent variable \(x\) as \(x(t)\) when entering Eq. (0.3). Otherwise, Maple evaluates the expression \texttt{diff(x, t)} as 0, because a general unknown \(x\) has no dependence on \(t\). The variable \texttt{bad} holds the relation \(0 = kx\), which is not a differential equation.

Exercises

1. If you define

\[ p := (x-4) * (x-5) * (x-6); \]

then the command

\[ dp := \text{diff}(p, x); \]

assigns a cumbersome expression to \(dp\). Show how to find an equivalent, simplified expression for the derivative.

2. Find the derivative with respect to \(t\) of each expression in the following list by hand. Then formulate and execute a Maple command to check your work.
(a) $\sin(2t - \delta)$  
(b) $e^{-2t}$  
(c) $2^{2t}$  
(d) $(2t - 3)^3$  
(e) $\frac{1}{(2t - 3)^2}$

3. Exhibit a sequence of Maple commands to find all the relative maxima and minima of the polynomial $(x - 1)(x - 2)(x - 3)$.

4. Exhibit a sequence of Maple commands to enter in each of the differential equations in the following list.

(a) $x' = x + 1$  
(b) $x'' + x = 0$  
(c) $x' = x^2$

0.8 Integration

Maple allows you to perform definite and indefinite integration symbolically. The following session calculates the antiderivative for the function $f(x) = \sin^3 x \cos^2 x$ and its definite integral over the interval $[0, \pi]$. Note that Maple does not include the constant of integration when it evaluates an indefinite integral.

> $f := \sin(x)^3 * \cos(x)^2;$

\[
f := \sin(x)^3 \cos(x)^2
\]

> int(f, x);

\[
\int \frac{1}{5} \sin(x)^3 \cos(x)^3 - \frac{2}{15} \cos(x)^3
\]

> int(f, x = 0 .. Pi);

\[
\frac{4}{15}
\]

If Maple cannot find a closed-form expression for a given integral, then it returns an unevaluated integral. Numerical approximations of unevaluated definite integrals often can be found using the evalf command, as the following session illustrates.

> int(ln(x)*exp(x^2), x = 1 .. 2);

\[
\int_1^2 \ln(x) e(x^2) \, dx
\]

> evalf(");

\[
8.057183539
\]
0.8. INTEGRATION

Exercises

1. Find an antiderivative of each of the following functions.
   (a) \( e^{-x} \sin x \)
   (b) \( t^3 e^{3t} \)
   (c) \( \tan t e^{2t} \)
   (d) \( \frac{x - 2}{x^2 + x - 12} \)

2. Find the definite integrals over the interval \([0, 1]\) for the functions in Exercise 1.

3. Determine the values of \(a\) and \(b\) for which Maple can evaluate the integral
   \[
   \int_a^b \frac{\sin(x)}{\ln(x)} \, dx.
   \]

   Does Maple behave as expected?

Command summary

The following is a list of the most important commands that are discussed in this chapter, together with some examples. For more details, consult the relevant subsection in the text, the Maple help facility, or a Maple reference manual.

" Refers to the most recently computed result. For instance, the following command sequence stores 12 into \(x\):

\[
4 \times 3; \\
x := ";
\]

\texttt{diff} Differentiates the indicated expression with respect to the specified variables.

For example, the following commands find the derivative of \((t^2 + 3t + 1)^{10}\):

\[
\text{fcn} := (t^2 + 3t + 1)^{10}; \\
\text{diff(fcn, t)};
\]

Either of the following commands finds the second derivative of the same expression:

\[
\text{diff(fcn, t, t)}; \text{ or } \text{diff(fcn, t$2$)};
\]

The following command assigns to \texttt{deq} the differential equation \(x' = x + 1\):

\[
\text{deq} := \text{diff}(x(t), t) = x(t) + 1;
\]

\texttt{evalf} Finds a floating-point approximation to the value of the indicated expression.

For example,

\[
\text{evalf}(2*\text{Pi});
\]
returns an approximation to $2\pi$.

**expand** Expands a given expression. It is often used with simplify to find a more convenient representation. For instance, the command

```maple
expand(");
```

expands the result of the previous Maple command.

**fsolve** Finds a numerical (floating-point) approximation to the solution of a given equation. The second argument gives the interval in which fsolve should seek a solution.

The following commands generate a floating-point approximation to $\sqrt{2}$:

```maple
p := t^2 - 2;
fsolve(p = 0, t = 0 .. 2);
```

**help** To invoke the Maple help facility, type

```maple
?topic
```

where topic refers to the command of interest.

**int** Integrates the indicated expression with respect to the specified variables. For instance, the following commands find the antiderivative of $t^2 + 3t + 1$:

```maple
f := t^2 + 3*t + 1;
in(f, t);
```

**lhs** Returns the left-hand side of the given equation.

**plot** Plots a graph of functions of a single variable. Multiple functions are enclosed in curly braces. For instance, the following commands plot the polynomial $t^2 + 3t + 1$ for $-5 \leq t \leq 5$:

```maple
p := t^2 + 3*t + 1;
plot(p, t = -5 .. 5);
```

The following command plots $e^t$ and $t^2$ on the same axes for $-1 \leq t \leq 1$:

```maple
plot({exp(t), t^2}, t = -1 .. 1);
```

**rhs** Returns the right-hand side of the given equation.

**simplify** Applies various identities to reduce an expression to a simpler one. For instance, the following command tries to simplify the output of the previous Maple command:
simplify(" ");

solve Attempts to find an exact (analytic) expression for the solution of a given equation. The second argument gives the variable to be solved for. For instance, the following command generates $\sqrt{2}$ and $-\sqrt{2}$:

\begin{verbatim}
p := t^2 - 2;
solve(p = 0, t);
\end{verbatim}

subs Substitutes a value or expression for the indicated variable or variables. For instance, the following commands evaluate the polynomial $x^3 + 3x + 2$ at $x = 3$:

\begin{verbatim}
p := x^3 + 3*x + 2;
subs(x=3, p);
\end{verbatim}

The following commands evaluate the polynomial $ax^2 + 6x + 2$ for $a = 1$ and $x = 4$:

\begin{verbatim}
p := a*x^2 + 6*x + 2;
subs(a=1, x=4, p);
\end{verbatim}