Each problem is worth 13 points. Explain, justify, and/or illustrate your method of solution. If Maple is used to find derivatives, state this and write down the results.

1. Find and classify the extrema of \( f(x, y) = 2x^3 - 3x^2 - 12x + 2y^3 - 54y \)

\[
\begin{align*}
\frac{\partial f}{\partial x} &= 6x^2 - 6x - 12 \\
\frac{\partial f}{\partial y} &= 6y^2 - 54 \\
6(x^2 - x - 2) &= 0 \\
(x - 2)(x + 1) &= 0 \\
x &= -1, 2 \\
6(y^2 - 9) &= 0 \\
y &= \pm 3 \\
\end{align*}
\]

\[
\begin{align*}
\frac{\partial^2 f}{\partial x^2} &= 12x - 6 \\
\frac{\partial^2 f}{\partial y^2} &= 12y \\
\frac{\partial^2 f}{\partial x \partial y} &= 0 \\
0 &= (12x - 6)12y - 0 \\
\end{align*}
\]

\((-1, 3): D < 0, S.P.
\(-1, -3): D > 0, \frac{\partial^2 f}{\partial x^2} < 0, \text{ REL. MAX.}
(2, 3): D > 0, \frac{\partial^2 f}{\partial x^2} > 0, \text{ REL. MIN.}
(2, -3): D < 0, S.P.
2. Find and classify the extreme points of \( f(x, y) = x^4 - xy + y^4 \)

\[
\begin{align*}
\frac{f_x}{f_y} &= 4x^3 - y \\
&= 4y^3 - x \\
y &= 4x^3 \\
y &= 4(y^3)^3 \\
y &= 456y^9 \\
y = 357y^8 - 1 = 0 \\
y = 0, y = \pm \frac{1}{2} \\
x = 0, x = \pm \frac{1}{2}, -\frac{1}{2} \\
\end{align*}
\]

\[
\begin{align*}
\frac{f_{xx}}{f_{yy}} &= 12x^2 \\
&= 12y^2 \\
\frac{f_{xy}}{f_{yx}} &= -1 \\
0 &= 12x^2 (12y^2) - (-1)^2 \\
(0, 0): &\quad 0 < 0 \Rightarrow s.p. \\
(\pm \frac{1}{2}, \pm \frac{1}{2}): &\quad 0 > 0, f_{xx} > 0, \text{ rel. min.} \\
\end{align*}
\]
3. Find and classify the critical points of \( f(x, y) = x^2 + 2y^2 - x^2 y \)

\[
f_x = 2x - 2xy \quad \quad \quad \quad f_y = 4y - x^2
\]

\[
2x(1-y) = 0 \quad \quad \quad \quad 4y = x^2
\]

\[
x = 0, \quad y = 1
\]

\[
y = 0, \quad x = \pm 2
\]

\[
f_{xx} = 2 - 2y \quad \quad f_{xy} = 4 \quad \quad f_{yy} = -2x
\]

\[
D = (2 - 2y)^2 - (-2x)^2
\]

\((0, 0)\) : \(D > 0, \quad f_{xx} > 0, \quad \text{rel. min.}\)

\((2, 1)\) : \(D < 0, \quad \text{s.p.}\)

\((-2, 1)\) : \(D < 0, \quad \text{s.p.}\)
4. A dietician wished to determine if a linear relationship exist between the height of a female and her weight. The table below gives the heights and weights of 9 females aged 18–24. The heights are measured in centimeters and the weights are measured in kilograms. Find a linear regression equation which could be used to predict the weight of a female given her height and use it to predict the weight of a woman whose height is 185 cm.

<table>
<thead>
<tr>
<th>Height</th>
<th>152</th>
<th>157</th>
<th>162</th>
<th>165</th>
<th>170</th>
<th>173</th>
<th>175</th>
<th>178</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>47</td>
<td>50</td>
<td>52</td>
<td>54</td>
<td>56</td>
<td>58</td>
<td>61</td>
<td>63</td>
<td>65</td>
</tr>
</tbody>
</table>

Formulas (if needed):

\[
b = \frac{\sum x^2 \sum y - (\sum x)(\sum xy)}{n \sum x^2 - (\sum x)^2}
\]

\[
m = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}
\]

\[
y = -48.33 + .622x
\]

USE MAPLE OR A CALCULATOR

\[
y = -48.33 + .622(185)
\]

66.74 kg
5. Suppose a post office will not mail a rectangular box if the sum of its length and girth (the perimeter of a cross section that is perpendicular to the length) is more than 108 inches. Find the dimensions of the box of maximum volume that can be mailed.

\[ V = lwh \]
\[ 108 = l + 2w + 2h \]

\[ \nabla V = \nabla G \]
\[ (wh, lh, lw) = \lambda (1, 2, 2) \]

\[ wh = \lambda \]
\[ lh = 2\lambda \]
\[ lw = 2\lambda \]

\[ \frac{wh}{2} = \lambda \]
\[ \frac{lh}{2} = \lambda \]
\[ \frac{lw}{2} = \lambda \]

\[ \frac{lh}{2} = \frac{lw}{2} \]

\[ l = w \]

\[ wh = \frac{lh}{2} \]

\[ l = 2w \]

\[ 2w + 2w + 2w = 108 \]

\[ 6w = 108 \]
\[ w = 18 \]
\[ h = 18 \]
\[ l = 36 \]

\[ V(18, 18, 36) = 11664 \]
\[ V(26, 26, 4) = 2704 \]

\[ \therefore \text{MAX.} \]
6. Optimize \( f(x, y) = xy + yz + xz \) subject to the constraint \( x + y + z = 1 \).

\[
\nabla f = \lambda \nabla g
\]

\[
(y + z, x + z, x + y) = \lambda (1, 1, 1)
\]

\[
y + z = x + z = x + y = \lambda
\]

\[
y + z = x + z \quad y + z = x + y
\]

\[
y = z \quad z = x
\]

\[
x = y = z
\]

\[
f \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) = \frac{1}{3}
\]

\[
f \left( \frac{1}{3}, \frac{1}{3}, 0 \right) = \frac{1}{4}
\]

\[
\therefore \text{ Maximum}
\]