Problems Part 5

1. A car rode over an ant on the pavement. The ant stuck to the tire for one revolution and then was deposited back onto the pavement. Assuming the radius of the tire is one foot, find the length of the curve traveled by the ant between its death and its final resting place.

2. A dart, thrown at random, hits a square target. Assuming that any two parts of the square with the same area are equally likely to be hit, find the probability that the point hit is nearer to the center than to any edge.

3. A 2×3 rectangle has vertices as (0, 0), (2, 0), (0, 3), and (2, 3). It rotates 90° clockwise about the point (2, 0). It then rotates 90° clockwise about the point (5, 0), then 90° clockwise about the point (7, 0), and finally, 90° clockwise about the point (10, 0). (The side originally on the x-axis is now back on the x-axis.) Find the area of the region above the x-axis and below the curve traced out by the point whose initial position is (1,1).

4. Find, with explanation, the maximum value of \( f(x) = x^3 - 3x \) on the set of all real numbers \( x \) satisfying \( x^4 + 36 \leq 13x^2 \).

5. Evaluate

\[
\int_2^4 \frac{\sqrt{\ln(9-x)}}{\sqrt{\ln(9-x) + \sqrt{\ln(x+3)}}} \, dx.
\]

6. Find all real-valued continuously differentiable functions \( f \) on the real line such that for all \( x \),

\[
(f(x))^2 = \int_0^x [(f(t))^2 + (f'(t))^2] \, dt + 1990.
\]