Problems Part 2

1. (a) A party is held at the house of Mr. and Mrs. A. There are three other couples present: B’s, C’s, and D’s. Some people shook hands. Nobody shook hands with anybody twice, and nobody shook hands with his/her spouse. At the end of the party, Mr. A interviewed each person (other than himself) how many hands they shook. Each person gave a different answer. Determine how many hands Mrs. A has shaken.

(b) Answer the same question with $n$ other couples instead of three.

2. (a) Consider the sequence of positive integers: 1, 12, 123, 1234, …, where the next term is constructed by lengthening the previous term at its right-hand end by appending the next positive integer. Note that this next integer occupies only one place, with “carrying” occurring as in addition. Thus the ninth and tenth terms of the sequence are 123,456,789 and 1,234,567,900, respectively. Determine which terms of the sequence are divisible by 11.

(b) Try to answer the same question with the number 11 replaced by a different prime number — say 5, or 7, etc.

3. In what base $b$ is $221_b$ a factor of $1215_b$?

4. Find all three digit numbers $N$ such that $N$ is evenly divisible by 11 and the sum of the squares of the (decimal) digits of $N$ is $N/11$.

5. Find the prime factorization of 1,006,015,020,015,006,001.