Relying on (2), we can prove (i) in Theorem 6.2.1. It suffices to prove that

\[ A < 0 \quad \text{and} \quad AC-B^2 > 0 \implies Ah^2 + 2Bhk + Ck^2 < 0 \quad \text{for all} \quad (h, k) \neq (0, 0) \]  

(3)

To this end we "complete the square":

\[ Ah^2 + 2Bhk + Ck^2 = A\left( \frac{h+Bk}{A} \right)^2 + \frac{AC-B^2}{A^2}k^2 \]  

(4)

The square bracket is obviously \( \geq 0 \), and equals 0 only if both \( h+Bk/A = 0 \) and \( k = 0 \), implying that \( h = k = 0 \). Because \( A < 0 \), the whole expression in (4) is negative for all \( (h, k) \neq (0, 0) \), so we have proved (2).

PROBLEMS SET FOR SECTION 6.2

1. Find the critical points on the following surfaces and determine whether each is a maximum, minimum or saddle point.

(a) \( f(x, y) = x^2 + xy + 3x + 2y + 2 \)

(b) \( f(x, y) = 2xy - 5x^2 - 2y^2 + 4x - 8 \)

(c) \( f(x, y) = x^2 + 3xy + 4y^2 - 5x + 2y \)

(d) \( f(x, y) = x^2 + xy + y^2 + x - 4y + 4 \)

(e) \( f(x, y) = x^2 - y^2 - 2xy + 3 \)

(f) \( f(x, y) = 6x^2 - 2x^2 + 3y^2 + 6xy \)

(g) \( f(x, y) = e^{xy} \)

(h) \( f(x, y) = e^{x^2 + y^2} \)

(i) \( f(x, y) = xy + \frac{x}{y} \)

2. FPM, Inc., owned by the Fabulous Miss Pepper Mills, is the biggest competitor of Slatina Jeans, Inc. They produce the same two types of jeans, regular and with lyca. The weekly cost function for FMP, Inc., is given by the function \( C(x, y) = 200 - 12x - 10y + .03x^2 + \frac{1}{3000}y^2 \) where \( x \) is the number of pairs of regular jeans and \( y \) is the number of pairs jeans with lyca. Miss Pepper hears about your good reputation as a consultant and hires you to determine how many pairs of each type of jeans they should produce weekly in order to minimize their costs.

3. Given the fierce competition from FMP, Inc., Slatina Jeans opened up another factory and hired the famous designers Tekka and 1t0d0e to work for them. At this factory they produce T-shirts and tank tops. The weekly profit equation for this factory is given by \( P(x, y) = 80x + 60y - 0.03x^2 - 0.03y^2 - 0.01xy - 4000 \) where \( x \) is the number of