Solution: The stationary points must satisfy the two equations

\[ f_1(x, y) = 3x^2 - 2x = 0 \quad \text{and} \quad f_2(x, y) = -2y = 0 \]

Because \(3x^2 - 2x = x(3x - 2)\), we see that the first equation has the solutions \(x = 0\) and \(x = 2/3\). The second equation has the solution \(y = 0\). We conclude that \((0, 0)\) and \((2/3, 0)\) are the only stationary points.

Furthermore,

\[ f_{xx}(x, y) = 6x - 2, \quad f_{xy}(x, y) = 0, \quad \text{and} \quad f_{yy}(x, y) = -2 \]

A convenient way of classifying the stationary points is to make a table like the following (with \(A, B,\) and \(C\) defined in the theorem):

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(AC - B^2)</th>
<th>Type of stationary point:</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, 0))</td>
<td>-2</td>
<td>0</td>
<td>-2</td>
<td>4</td>
<td>Local maximum point</td>
</tr>
<tr>
<td>((2/3, 0))</td>
<td>2</td>
<td>0</td>
<td>-2</td>
<td>-4</td>
<td>Saddle point</td>
</tr>
</tbody>
</table>

**Example 3**

Find the stationary points of \(f(x, y)\) and classify them when

\[ f(x, y) = \ln(x + y) - x^2 - y^2 + x, \quad x > 0, \quad y > 0 \]

Solution: The stationary points must satisfy the two equations

\[ f_1(x, y) = \frac{1}{x + y} - 2x + 1 = 0 \quad \text{and} \quad f_2(x, y) = \frac{1}{x + y} - 2y = 0 \quad (\ast) \]

It follows from these two equations that \(2x - 1 = 2y\), so \(x = y - 1/2\). Inserting this into the first equation of \((\ast)\), then rearranging, yields the second-degree equation \(4x^2 - 3x - \frac{1}{2} = 0\).

The only positive solution is \(x = \frac{1}{8}(3 + \sqrt{17})\) and then \(y = x - 1/2 = \frac{1}{8}(-1 + \sqrt{17})\).

Furthermore,

\[ f_{xx}(x, y) = -\frac{1}{(x + y)^2} - 2, \quad f_{xy}(x, y) = -\frac{1}{(x + y)^2}, \quad f_{yy}(x, y) = -\frac{1}{(x + y)^2} - 2 \]

It follows that

\[ A = f_{xx}(x, y), \quad B = f_{xy}(x, y), \quad C = f_{yy}(x, y) \]

It follows that

\[ f_{xx}(x, y) f_{yy}(x, y) - (f_{xy}(x, y))^2 = \frac{4}{(x + y)^2} + 4 \]

So according to Theorem 6.2.1 the only stationary point is a local maximum point. (In fact, Theorem 6.1.2 allows us to conclude that the stationary point is a global maximum.)