(c) What is the largest value of \( f(x, y) \) among those \((x, y)\) that satisfy \(2x + 3y = 12\) ?

22. Suppose \( F(x, y) \) is a function about which all we know is that \( F(0, 0) = 0 \), as well as that \( F_x(x, y) \geq 2 \) for all \((x, y)\), and \( F_y(x, y) \leq 1 \) for all \((x, y)\). What can be said about the relative sizes of \( F(1, 0) \), \( F(1, 0), F(2, 0), F(0, 1)\), and \( F(1, 1)\)? Write down the inequalities that have to hold between these numbers.

23. Consider the sphere \( x^2 + y^2 + z^2 = 4 \). If \( f(x, y) \) is the upper half of the sphere, decide what the signs will be of the following partial derivatives (Do this based on the shape of the upper hemisphere, do not compute the numeric value of the partial derivatives.)

(a) \( f_x(1, 1) \)  
(b) \( f_y(1, 1) \)  
(c) \( f_x(-1, 1) \)  
(d) \( f_y(-1, 1) \)  
(e) \( f_x(-1, -1) \)  
(f) \( f_y(-1, -1) \)  
(g) \( f_x(1, -1) \)  
(h) \( f_y(1, -1) \)

24. Consider the sphere \( x^2 + y^2 + z^2 = 4 \). If \( f(x, y) \) is the lower half of the sphere, decide what the signs will be of the following partial derivatives (Do this based on the shape of the lower hemisphere, do not compute the numeric value of the partial derivatives.)

(a) \( f_x(1, 1) \)  
(b) \( f_y(1, 1) \)  
(c) \( f_x(-1, 1) \)  
(d) \( f_y(-1, 1) \)  
(e) \( f_x(-1, -1) \)  
(f) \( f_y(-1, -1) \)  
(g) \( f_x(1, -1) \)  
(h) \( f_y(1, -1) \)

5.5 Functions of More Variables

Many of the most important functions we study in business and economics, such as the gross domestic product (GDP) of a country, depend on a very large number of variables. In some abstract models, it may be sufficient to ascertain that such a connection exists without specifying the dependence more closely. In this case, we say only that the GDP is a function of the different variables.