Solution:

(a) If you stand at $P$, you are on the level curve $f(x, y) = 2$. If you look in the direction of the positive $x$-axis (along the line $y = 4$), then you will see the terrain sloping upwards, because the (nearest) level curves will correspond to larger $z$ values. Hence, $f_x > 0$. If you stand at $P$ and look in the direction of the positive $y$-axis (along $x = 2$), the terrain will slope downwards. Thus, at $P$, we must have $f_y < 0$. At $Q$, we find similarly that $f_x < 0$ and $f_y > 0$. To estimate $f_x(3, 1)$, we use $f_x(3, 1) = f(4, 1) - f(3, 1) = 2 - 4 = -2$. This approximation is actually far from exact. If we keep $y = 1$ and decrease $x$ by one unit, then $f(2, 1) \approx 4$, which should give the estimate $f_x(3, 1) \approx 4 - 4 = 0$. "The map" is not sufficiently finely graded around $Q$.

(b) Equation (i) has the solutions $y = 1$ and $y = 4$, because the line $x = 3$ cuts the level curve $f(x, y) = 4$ at $(3, 1)$ and $(3, 4)$. Equation (ii) has no solutions, because the line $y = 4$ does not meet the level curve $f(x, y) = 0$ at all.

(c) The highest value of $c$ for which the level curve $f(x, y) = c$ has a point in common with the line $x = 2$ is $c = 6$. The largest value of $f(x, y)$ when $x = 2$ is therefore $6$, and we see from Fig. 10 that this maximum value is attained when $y \approx 2.2$.

PROBLEMS SET FOR SECTION 5.4

1. Find $\partial z/\partial x$ and $\partial z/\partial y$ for the following:
   
   \begin{align*}
   (a) & \quad z = 2x + 3y \\
   (b) & \quad z = x^2 + y^3 \\
   (c) & \quad z = x^2y^4 \\
   (d) & \quad z = (x + y)^2
   \end{align*}

2. Find $\partial z/\partial x$ and $\partial z/\partial y$ for the following:
   
   \begin{align*}
   (a) & \quad z = x^2 + 3y^2 \\
   (b) & \quad z = xy \\
   (c) & \quad z = 5x^4y^2 - 2xy^5 \\
   (d) & \quad z = e^{x+y}  \\
   (e) & \quad z = e^{xy} \\
   (f) & \quad z = e^x/y \\
   (g) & \quad z = \ln(x + y) \\
   (h) & \quad z = \ln(xy)
   \end{align*}

3. Find $f_x(x, y)$, $f_y(x, y)$, and $f_{xy}(x, y)$ for the following:
   
   \begin{align*}
   (a) & \quad f(x, y) = x^2 - y^2 \\
   (b) & \quad f(x, y) = x^2 \ln y \\
   (c) & \quad f(x, y) = (x^2 - 2y^2)^2
   \end{align*}