Solution: We find that \( \partial x / \partial p = -1.5A p^{-2.5} m^{-0.8} \) and \( \partial x / \partial m = 2.08A p^{-1.5} m^{1.08} \). Because \( A, p, \) and \( m \) are positive, \( \partial x / \partial p < 0 \) and \( \partial x / \partial m > 0 \). These signs are in accordance with the final remarks in the example.

Formal Definitions of Partial Derivatives

So far the functions have been given by explicit formulas and we have found the partial derivatives by using the ordinary rules for differentiation. If these rules cannot be used, however, we must resort directly to the formal definition of partial derivative. This is derived from the definition of derivative for functions of one variable in the following rather obvious way.

If \( z = f(x, y) \), then with \( g(x) = f(x, y) \) (y fixed), the partial derivative of \( f(x, y) \) w.r.t. \( x \) is simply \( g'(x) \). Now, the definition \( g'(x) \) is \( g'(x) = \lim_{h \to 0} [g(x + h) - g(x)]/h \). Because \( f'_x(x, y) = g'(x) \), it follows that

\[
f'_x(x, y) = \lim_{h \to 0} \frac{f(x + h, y) - f(x, y)}{h} \tag{2}
\]

In the same way,

\[
f'_y(x, y) = \lim_{k \to 0} \frac{f(x, y + k) - f(x, y)}{k} \tag{3}
\]

If the limit in (2) (or (3)) does not exist, then we say that \( f'_x(x, y) \) (or \( f'_y(x, y) \)) does not exist, or that \( z \) is not differentiable w.r.t. \( x \) or \( y \) at the point. For instance, the function \( f(x, y) = |x| + |y| \) is not differentiable at the point \( (x, y) = (0, 0) \).

If \( h = dx \) is small in absolute value, then from (2) we obtain the approximation, \( f'_x(x, y) \approx [f(x + dx, y) - f(x, y)]/dx \), so

\[
f'_x(x, y)dx \approx f(x + dx, y) - f(x, y) \tag{4}
\]

Similarly, if \( dy \) is small in absolute value,

\[
f'_y(x, y)dy \approx f(x, y + dy) - f(x, y) \tag{5}
\]

In these approximations, if we put \( dx = 1 \) and \( dy = 1 \), we can interpret them in the following way:

(A) The partial derivative \( f'_x(x, y) \) is approximately equal to the change in \( f(x, y) \) that results from increasing \( x \) by one unit while holding \( y \) constant.

(B) The partial derivative \( f'_y(x, y) \) is approximately equal to the change in \( f(x, y) \) that results from increasing \( y \) by one unit while holding \( x \) constant.