CHAPTER 5 / FUNCTIONS OF SEVERAL VARIABLES

$KL$-plane given by

$$F(K, L) = Y_0 \quad (Y_0 \text{ is a constant})$$

This curve is called an isoquant (indicating "equal quantity"). For a Cobb-Douglas function $F(K, L) = AK^aL^b$ with $a + b < 1$ and $A > 0$, Figs. 7 and 8 respectively show a part of the graph near the origin, and three of the isoquants.

![Figure 7 and Figure 8](image)

**EXAMPLE 3** Show that all points $(x, y)$ satisfying $xy = 3$ lie on a level curve for the function

$$g(x, y) = \frac{3(xy + 1)^2}{x^2y^2 - 1}$$

**Solution:** By substituting $xy = 3$ in the expression for $g$, we find

$$g(x, y) = \frac{3(3 + 1)^2}{(3)^2 - 1} = \frac{3(4)^2}{3^2 - 1} = \frac{48}{80} = \frac{3}{5}$$

This shows that, for all $(x, y)$ where $xy = 3$, the value of $g(x, y)$ is a constant $3/5$. Hence, any point $(x, y)$ satisfying $xy = 3$ is on a level curve (at height $3/5$) for $g$. (In fact, for any value of $c \neq \pm 1, xy = c$ is the equation of a level curve for $g$ because $g(x, y) = 3(c+1)^2/(c^2 - 1)$ when $xy = c \neq \pm 1$.)

**PROBLEMS SET FOR SECTION 5.3**

1. Draw a three-dimensional coordinate system and mark the points

$$P = (3, 0, 0), \quad Q = (0, 2, 0), \quad R = (0, 0, -4), \quad S = (3, -2, 4)$$

(For $S$, you should draw a box like those in Figs. 1 and 2.)