Figure 3  Graph of \( y = f(x, y) \).  \( P = (x_0, y_0, f(x_0, y_0)) \)

This method of representing a function of two variables helps us to visualize its behaviour in broad outline. However, it requires considerable artistic ability to represent in only two dimensions the graph of \( z = f(x, y) \), which lies in three-dimensional space.  It is certainly difficult to use the resulting drawing for quantitative measurements.  (Using modern computer graphics, however, complicated functions of two variables can have their graphs drawn fairly easily, and these can be rotated or transformed to display the shape of the graph better.)

We now describe a second kind of geometric representation that often does better when we are confined to two dimensions (as we are in the pages of this book).

**Level Curves**

Map makers can describe some topographical features of the earth's surface such as hills and valleys even in a plane map.  One way of doing so is to draw a set of **level curves** or contours connecting points on the map that represent places on the earth's surface with the same elevation above sea level.  For instance, there may be such contours corresponding to 100 metres above sea level, others for 200, 300, and 400 metres above sea level, and so on.  Off the coast, or in places like the valley of the River Jordan, there may be contours for 100 metres below sea level, etc.  Where the contours are close together, there is a steep slope.  Thus, studying a contour map carefully can give a good idea how the elevation varies on the ground.

The same idea can be used to give a geometric representation of an arbitrary function \( z = f(x, y) \).  The graph of the function in three-dimensional space is visualized as being cut by horizontal planes parallel to the \( xy \)-plane.  The resulting intersection between each plane and the graph is then projected onto the \( xy \)-plane.  If the intersecting plane is \( z = c \), then the projection of the intersection onto the \( xy \)-plane is called the **level curve** at height \( c \) for \( f \).  This level curve will consist of points satisfying the equation

\[
f(x, y) = c
\]

Figure 4 illustrates the construction of such a level curve.