Recall how any point in a plane can be represented by a pair of real numbers by using two mutually orthogonal coordinate lines: a rectangular coordinate system in the plane. In a similar way, points in space can be represented by triples of real numbers using three mutually orthogonal coordinate lines. In Fig. 1 we have drawn such a coordinate system. The three lines that are orthogonal to each other and intersect at the point \( O \) in Fig. 1 are called coordinate axes. They are usually called the \( x \)-axis, \( y \)-axis, and \( z \)-axis. We choose units to measure the length along each axis, and select a positive direction on each of them as indicated by the arrows.

![Figure 1](image1.png)

![Figure 2](image2.png)

The equation \( x = 0 \) is satisfied by all points in a coordinate plane spanned by the \( y \)-axis and the \( z \)-axis. This is called the \( yz \)-plane. There are two other coordinate planes: the \( xy \)-plane on which \( z = 0 \), and the \( xz \)-plane on which \( y = 0 \). We often think of the \( xy \)-plane as horizontal, with the \( z \)-axis passing vertically through it.

Each coordinate plane divides the space into two half-spaces. For example, the \( xy \)-plane separates the space into the regions where \( z \geq 0 \), above the \( xy \)-plane, and \( z \leq 0 \), below the \( xy \)-plane. The three coordinate planes together divide the space into 8 octants. The octant which has \( x \geq 0, y \geq 0, \) and \( z \geq 0 \) is called the nonnegative octant.

Every point \( P \) in space now has an associated triple of numbers \((x_0, y_0, z_0)\) that describes its location, as suggested in Fig. 1. Conversely, it is clear that every triple of numbers also represents a unique point in space in this way. Note in particular that when \( z_0 \) is negative, the point \((x_0, y_0, z_0)\) lies below the \( xy \)-plane in which \( z = 0 \). In Fig. 2, we have constructed the point \( P \) with coordinates \((-2, 3, -4)\). The point \( P \) in Fig. 1 lies in the positive octant.

The Graph of a Function of Two Variables

Suppose \( z = f(x, y) \) is a function of two variables defined over a domain \( D \) in the \( xy \)-plane. The graph of the function \( f \) is the set of all points \((x, y, f(x, y))\) in the space obtained by letting \((x, y)\) "run through" \( D \). If \( f \) is a sufficiently "nice" function, the graph of \( f \) will be a connected surface in the space, like the graph in Fig. 3. In particular, if \((x_0, y_0)\) is a point in the domain \( D \), we see how the point \( P = (x_0, y_0, f(x_0, y_0)) \) on the surface is obtained by letting \( f(x_0, y_0) \) be the "height" of \( f \) at \((x_0, y_0)\).