Review for Exam 1, MAT 243

Think of this review as a starting point for studying for Exam 1.

1. Show that
\[(p \rightarrow q) \leftrightarrow (\neg p \lor q)\]
is a tautology.

2. Show without truth tables that the following logical equivalence holds:
\[(p \rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)\]

3. Construct the truth table for
\[[(p \rightarrow q) \land (q \rightarrow r)] \rightarrow (p \rightarrow r)\]

4. Translate these specifications into English where \(F(p)\) is “Printer \(p\) is out of service,” \(B(p)\) is “Printer \(p\) is busy,” \(L(j)\) is “Print job \(j\) is lost,” and \(Q(j)\) is “Print job \(j\) is queued.”
   (a) \(\exists p(F(p) \land B(p)) \rightarrow \exists jL(j)\)
   (b) \(\forall pB(p) \rightarrow \exists jQ(j)\)
   (c) \(\exists j(Q(j) \land L(j)) \rightarrow \exists pF(p)\)

5. Let \(A = \{1, 3, 5\}\) and \(B = \{x, y\}\).
   (a) What is the power set of \(A\)?
   (b) What is \(A \times B\)?

6. Explain which rules of inference are used for each step.

   Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket.

7. Show that the argument form with premises
   (a) \(p\),
   (b) \(p \rightarrow p \land r\),
   (c) \(s \rightarrow \neg r\)
and conclusion \( \neg s \) is valid. Identify which rules of inference you used.

8. Let \( A_i = [0, i) \).
   
   (a) Find
   \[
   \bigcup_{i=1}^{\infty} A_i
   \]
   and prove your claim.
   
   (b) Find
   \[
   \bigcap_{i=1}^{\infty} A_i
   \]
   and prove your claim.

9. (a) Let \( f : \mathbb{Z} \to \mathbb{Z} \) be the function \( f(x) = 5x - 4 \). Determine whether \( f \) is onto (surjective). \textit{Prove your claim.}
   
   (b) Let \( f : \mathbb{Z} \to \mathbb{Z} \) be the function \( f(x) = 3x - 1 \). Determine whether \( f \) is one-to-one (injective). \textit{Prove your claim.}

\textit{(Note that \( \mathbb{Z} \) denotes the set of integers, and \( \mathbb{R} \) the set of real numbers.)}

10. What is the cardinality of the following sets? (finite? countable? uncountable?)
   
   (a) \( \{ x \in \mathbb{R} | x/2 \text{ is an integer} \} \)
   
   (b) \( \{ x \in \mathbb{R} | x/2 \text{ is a positive integer less than 10} \} \)
   
   (c) \( \{ x \in \mathbb{R} | \lfloor x \rfloor = 0 \} \)

11. Evaluate
   \[
   \sum_{k=0}^{200} \left( \frac{1}{3} \right)^k
   \]
   Leave your answer as a fraction, do not simplify and do not evaluate exponents.

12. Show that \( \overline{A \cup B} = \overline{A} \cap \overline{B} \).