The exam is comprehensive. This review sheet contains only problems from 7.1, 7.2, 8.1, and 8.5. Use this review sheet, previous review sheets, exams, and homework as a starting point for studying.

1. (a) Find a recurrence relation for the number of ternary strings that contain two consecutive symbols that are the same.
   (b) What are the initial conditions?
   (c) How many ternary strings of length six contain consecutive symbols that are the same?

2. Find the solution of the recurrence relation \( a_n = 2a_{n-1} + 3 \cdot 2^n \).

3. Find all solutions of the recurrence relation \( a_n = 7a_{n-1} - 16a_{n-2} + 12a_{n-3} + n4^n \) with \( a_0 = -2 \), \( a_1 = 0 \), and \( a_2 = 5 \).

4. Let \( R \) be the relation \( R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a \leq b \} \).
   (a) Determine \( R^1 \), \( R^2 \), \( R^3 \), and \( R^k \).
   (b) Prove that your \( R^k \) is correct.

5. Suppose that \( R_1 \) and \( R_2 \) are reflexive relations on a set \( A \). Is \( R_1 \cap R_2 \) also reflexive? Is \( R_1 \cup R_2 \) also reflexive?

6. Determine whether the relation \( R \) on the set of all real numbers is reflexive, symmetric, and/or transitive, where \((x, y) \in R\) if and only if
   (a) \( x - y \) is a rational number.
   (b) \( xy \geq 1 \).
   (c) \( x \neq y \).
   (d) \( x \) is a multiple of \( y \).
   (e) \( x = y^2 \).
   (f) \( x \geq y^2 \).
7. Find the smallest equivalence relation on the set \{a, b, c, d, e\} containing the relation \{(a, b), (a, c), (d, e)\}.

8. (a) Show that the relation \(R\) consisting of all pairs \((x, y)\) such that \(x\) and \(y\) are bit strings of length three or more that agree in their first three bits is an equivalence relation on the set of all bit strings of length three or more.

(b) What are the equivalence classes of the relation?

9. Let \(R\) be the relation on the set of all subsets \(A\) of \(\mathbb{Z}\) such that \(ARB\) if and only if \(|A| = |B|\).

(a) Show that \(R\) is an equivalence relation.

(b) Describe the equivalence classes.