The course will closely follow the text, the first five chapters of which offer a review of classical material largely covered in MAT476 so it will be covered quickly. Distributions, weak solutions, Sobolev spaces, and functional analytic methods are encountered in the second half of the book, which will be the primary focus of the course. Homework exercises will be regularly assigned and the course grade will depend only on the homework.

Useful Texts & Monographs on 3-day reserve at Noble Library:

Homework: Due Oct 1.
# 5,7, p. 48.
# 3,8,10,13 p. 71.
# 4 p. 82.
If the support of $h, g$ is contained in $[-\epsilon, \epsilon]$ estimate the support of $u(x,t)$, as a function of $x$, for each $t > 0$ for solution (6) page 75 of text. If a person sits at position $x$, when will (give time interval) she observe the disturbance (non-zero values of $u$)?)
#1, 6 p. 90.
#2, 6 p. 94.
#3,6,9 p. 110.
#4,7, 11 p. 125.
Homework: Due Nov 12.
Let $0 < \alpha < \beta < 1$ and $\Omega \subset \mathbb{R}^n$ be a bounded domain. Show that $C^\alpha(\overline{\Omega})$ is a Banach space. Show that the two inclusions $C^\beta(\overline{\Omega}) \subset C^\alpha(\overline{\Omega}) \subset C(\overline{\Omega})$ are continuous imbeddings.

Homework: **Due no later than Dec. 13.**

Suppose $\Omega \subset \mathbb{R}^n$ is connected and $u \in W^{1,p}(\Omega)$ satisfies $\nabla u = 0$ almost everywhere in $\Omega$. Prove there is a constant $c$ such that $u = c$ almost everywhere. Hint: Mollify!

1. Given $f \in L^2(\Omega)$, let $u \in H^1_0(\Omega)$ be the unique weak solution of $\Delta u = f$ with $u = 0$ on $\partial \Omega$ given by our application of Riesz representation theorem in section 6.2. Write $u = Kf$. Show that $K : L^2(\Omega) \to H^1_0(\Omega)$ is a bounded linear operator. Show that this implies the well-posedness of the problem $\Delta u = f$ with $u = 0$ on $\partial \Omega$: existence, uniqueness, and continuous dependence on $f$. Hint: show $K$ is closed and use the closed graph theorem.

2. Let $D = H^1_0(\Omega) \cap H^2(\Omega)$ and $\Delta : D \subset L^2(\Omega) \to L^2(\Omega)$. Show that the Poincaré Inequality leads to the existence of $c > 0$ such that

\[ \langle -\Delta u, u \rangle \geq c\|u\|^2_2, \quad u \in D \]

(Use complex inner product!) Show that this implies that $N(\Delta) = 0$. Show that it implies $\lambda \leq -c$ for all eigenvalues of $\Delta$ (we already know $\sigma_p(\Delta) \subset \mathbb{R}$).

Homework: **Due no later than Mar. 13.**

MAT577 Spring 2005

Homework: **Due Monday May 9 by 9:00A.M.**
extra problem: If $f: [0, T] \to X$ is continuous and $\{S(t)\}_{t \geq 0}$ is a $C^0$-semigroup, show that $u(t) = \int_0^t S(t-s)f(s)ds$ is continuous on $[0, t]$.

extra problem: Consider the reaction-diffusion system

$$
\begin{align*}
    u_t & = \Delta u + u(u - 1)(2 - u), \quad x \in \Omega \\
    u(x, t) & = 0, \quad x \in \partial \Omega \\
    u(x, 0) & = g(x) \geq 0, \quad x \in \Omega
\end{align*}
$$

Assume that $g \in X_\alpha$, $1 \geq \alpha > n/4$, $n < 4$. By Example 3, pg 337, there is a $\tau > 0$ and a unique solution $u$ that exists for $(x, t) \in \overline{\Omega} \times [0, \tau]$. Let $T \leq \tau$.

(a) Show that $u \geq 0$.
(b) Show that $u(x, t) \leq v(t), (x, t) \in U_T$ where $v' = v(v - 1)(2 - v)$, $v(0) = \sup_{x \in \Omega} g$; conclude that the solution exists for all $t \geq 0$ and is bounded.
(c) Show that if $u(x)$ is a steady state or equilibrium solution, then $0 \leq u(x) \leq 2, \quad x \in \Omega$. Note that the initial condition $g$ is irrelevant in this case.
(d) Show that if either $\lambda_1 > \frac{1}{4}$ or $0 \leq g(x) < 1, \quad x \in \overline{\Omega}$, then $u(x, t) \to 0, \quad t \to \infty$ uniformly in $x \in \Omega$. Hint: $(u - 1)(2 - u) \leq 1/4$.

# 5(b), p. 339.