1.2 GAMES IN EXTENSIVE FORM

In our general idea of a game, therefore, three elements enter: (1) alternation of moves, which can be either personal or random (chance) moves, (2) a possible lack of knowledge, and (3) a payoff function.

We define, first, a topological tree or game tree as a finite collection of nodes, called vertices, connected by lines, called arcs, so as to form a connected figure which includes no simple closed curves. Thus it follows that, given any two vertices $A$ and $B$, there is a unique sequence of arcs and nodes joining $A$ to $B$.

From this we obtain

1.2.1 Definition Let $\Gamma$ be a topological tree with a distinguished vertex $A$. We say that a vertex $C$ follows the vertex $B$ if the sequence of arcs joining $A$ to $C$ passes through $B$. We say $C$ follows $B$ immediately if $C$ follows $B$ and, moreover, there is an arc joining $B$ to $C$. A vertex $X$ is said to be terminal if no vertex follows $X$.

1.2.2 Definition By an $n$-person game in extensive form is meant

(\alpha) a topological tree $\Gamma$ with a distinguished vertex $A$ called the starting point of $\Gamma$;

(\beta) a function, called the payoff function, which assigns an $n$-vector to each terminal vertex of $\Gamma$;

(\gamma) a partition of the nonterminal vertices of $\Gamma$ into $n + 1$ sets $S_0, S_1, \ldots, S_n$, called the player sets;

(\delta) a probability distribution, defined at each vertex of $S_0$, among the immediate followers of this vertex;

(\epsilon) for each $i = 1, \ldots, n$, a subpartition of $S_i$ into subsets $S_i^j$, called information sets, such that two vertices in the same information set have the same number of immediate followers and no vertex can follow another vertex in the same information set;

(\zeta) for each information set $S_i^j$, an index set $I_i^j$, together with a $1 - 1$ mapping of the set $I_i^j$ onto the set of immediate followers of each vertex of $S_i^j$.

The elements of a game are seen here: condition $\alpha$ states that there is a starting point; $\beta$ gives a payoff function; $\gamma$ divides the moves into chance moves ($S_0$) and personal moves which correspond to the $n$ players ($S_1, \ldots, S_n$); $\delta$ defined a randomization scheme at each chance move; $\epsilon$ divides a player's moves into "information sets": he knows which information set he is in, but not which vertex of the information set.
Figure 4.2. Structure of an Evolutionary Agent-Based Simulation
Figure 4.3. Structure of Reproduction Process