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# A Note of Welcome

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A note of welcome to the new *Journal of Graph Theory* might contain all sorts of good wishes and superficial praises of the beauty and usefulness of graph theory in general terms. My views on the latter, supported by facts, were given in [2]. As to the former, I can illustrate it better by giving some indications of the enchantment and help it gave me in the most difficult times of my life during the war.

It sounds a bit incredible but it is true. The story goes back to 1940 when I received a letter from Shanghai from my friend George Szekeres in which he described an unsuccessful attempt to prove a famous Burnside conjecture (which was disproved later). The failure of his attempt could be effected by a special case of Ramsey's theorem (but Ramsey's paper, beyond mere existence, was unknown at that time in Hungary). At that time my main financial income came from private tutoring, and I had to teach the pupils at their homes. After receiving the letter, and while traveling between two consecutive pupils, I was pondering on its content. The chain of thought soon led me to finite forms and then to the following extremal problem: What is the maximum number of edges in a graph with $n$ vertices not containing a complete subgraph with $k$ vertices? Though I found the problem definitely interesting, I postponed it, having been interested at that time mainly in problems in analytical number theory. In September 1940 I was called in for the first time to labor-camp service. We were taken to Transylvania to work at railway building. Our main work was carrying railway ties. It was not very difficult work but a spectator could of course easily recognize that most of us—I was no exception—did it rather awkwardly. One of my more expert comrades said this at one occasion quite explicitly, even mentioning my name. An officer was standing nearby, watching our work. When hearing my name, he asked the comrade whether or not I was a mathematician. It turned out that the officer—Joseph Winkler by name— was an engineer. In his youth he had placed at a mathematical competition; in civilian life he was a proofreader at the printing shop where the periodical of the Third Class of the Academy (Mathematical and Natural Sciences) was printed and

* Deceased 26 September 1976. The next issue of the *Journal of Graph Theory* will be dedicated to the memory of Paul Turán.

had seen some of my manuscripts. He could do no more than assign me to a wood-yard where big logs, necessary to railroad building, were stored, classified according to their diameter; my task was merely to show incoming groups the place where they could find those logs with the prescribed width. This was not as bad in the nice scenery, in the quite unpolluted air; I was outside walking the whole day. The problems I worked on in August returned to my mind, but I could not use paper to try out in detail my ideas as to whether or not they worked. Then the formal extremal problem occurred to me and I immediately felt that here was the problem appropriate to the circumstances. I cannot properly describe my feelings during the next few days. The pleasure of dealing with a quite unusual type of problem, the beauty of it, the gradual nearing of the solution, and finally the complete solution made these days really ecstatic. The feeling of some intellectual freedom and being, to a certain extent, spiritually free of oppression only added to this ecstasy.

My next encounter with graph theory in these years was of a quite different nature. In July 1944 the danger of deportation was real in Budapest, and a reality outside Budapest. We worked near Budapest, in a brick factory. There were some kilns where the bricks were made and some open storage yards where the bricks were stored. All the kilns were connected by rail with all the storage yards. The bricks were carried on small wheeled trucks to the storage yeards. All we had to do was to put the bricks on the trucks at the kilns, push the trucks to the storage yards, and unload them there. We had a reasonable piece rate for the trucks, and the work itself was not difficult; the trouble was only at the crossings. The trucks generally jumped the rails there, and the bricks fell out of them; in short this caused a lot of trouble and loss of time which was rather precious to all of us (for reasons not to be discussed here). We were all sweating and cursing at such occasions, I too; but *nolens-volens* the idea occurred to me that this loss of time could have been minimized if the number of crossings of the rails had been minimized. But what is the minimum number of crossings? I realized after several days that the actual situation could have been improved, but the exact solution of the general problem with $m$ kilns and $n$ storage yards seemed to be very difficult and again I postponed my study of it to times when my fears for my family would end. (But the problem occurred to me again not earlier than 1952, at my first visit to Poland where I met Zarankiewicz. I mentioned to him my "brick-factory"-problem; he mentioned to me another one referring to (0, 1)-matrices. We solved with T. Kovari and my wife his problem (while independently P. Erdös also solved this problem) and Zarankiewicz thought to have solved mine. But Ringel found a gap in his published proof, which nobody has been able to fill so far—in spite of much effort. This problem has also become a notoriously difficult unsolved problem; the present state of it and the ensuing general problems one can see in the interesting paper of Guy [1].)

But the greatest help I got from graph theory came in October and November 1944. As to our situation then, in short we did not have work to do but expected every day to be entrained, deported to the West. Still back in 1941, when I was discussing my graph results with my late friend Geza Grünwald, he raised the question (again not knowing Ramsey's paper): What is the greatest $M(n)$ such that for every graph $G$ with $n$ vertices either $G$ or $\bar{G}$ contains a complete subgraph with $M(n)$ vertices? This beautiful question came back to my memory at the end of October 1944 and I worked on it as much as I could, with full intensity until the middle of November. I had the idea that the extremal graph of this problem could be obtained, roughly speaking, by dividing the vertices into $[\sqrt{n}]$ disjoint classes (possibly equal) and connecting two vertices if and only if they belong to different classes. I still have the copybook in which I wrote down the various approaches by induction; they all started promisingly but broke down at various points. I had no other support for the truth of this conjecture than the symmetry and some dim feeling of beauty; perhaps the ugly reality was what made me believe in the strong connection of beauty and truth. But this unsuccessful fight gave me strength hence, when it was necessary, I could act properly. In one of my first letters to Erdös after the war I wrote of this conjecture to him. In his answer he proved that my conjecture was utterly false; $M(n)$ is about $\log n$—much less than the conjectured value (i.e., about $\sqrt{n}$). His disproof was only a proof of existence; perhaps this was the first time he applied probabilistic methods in graph theory. As far as I know no *explicit* graph is known to date for which neither the graph itself nor the complementary graph contains a complete subgraph with at least $n$ vertices.

The list is far from being complete but this note is perhaps already a bit too long for a welcome note. So I close it by expressing my wish and hope that I shall be able to read among many other important results the solution of the abovementioned open problems as well in this periodical.

REFERENCES

1. R. K. Guy, The decline and fall of Zarankiewicz's theorem. *Proof Techniques in Graph Theory*, Academic Press, New York (1969) 63–69.
2. P. Turán, Applications of graph theory to geometry and potential theory. *Combinatorial Structures and Theory Applications*, Gordon and Breach, New York (1970) 423–434.