Based on Strang’s *Introduction to Applied Mathematics*

**CG Algorithm**

To solve $Ax = b$ with $A$ symmetric and positive definite (if not, multiply $Ax = b$ by $A^T$), with residual $r = b - Ax$:

- initial guess $x_0 = 0$ and initial residual $r_0 = b$ (steepest descent)
- $\beta_j = r_{j-1}^T r_{j-1} / (r_{j-2}^T r_{j-2})$ except $\beta_1 = 0$ to insure $d_j^T Ad_i = 0$, $i \neq j$
- direction $d_j = r_{j-1} + \beta_j d_{j-1}$ except $d_1 = r_0$
- $\alpha_j = r_{j-1}^T r_{j-1} / (d_j^T Ad_j)$
- new guess $x_j = x_{j-1} + \alpha_j d_j$
- new residual $r_j = r_{j-1} - \alpha_j Ad_j$ with $r_j^T r_i = 0$, $i \neq j$

In exact arithmetic, $x_n = x$.

**PCG Algorithm**

To speed up convergence, precondition: $M^{-1}Ax = M^{-1}b$ with $M$ symmetric positive definite (below $S = M^{-1/2} AM^{-1/2}$). A popular choice for $M$ is the incomplete Cholesky factorization of $A$.

- initial guess $x_0 = 0$ and initial residual $r_0 = b$ (steepest descent)
- Solve $M z_{j-1} = r_{j-1}$
- $\beta_j = z_{j-1}^T r_{j-1} / (z_{j-2}^T r_{j-2})$ except $\beta_1 = 0$ to insure $d_j^T S d_i = 0$, $i \neq j$
- direction $d_j = z_{j-1} + \beta_j d_{j-1}$ except $d_1 = z_0$
- $\alpha_j = z_{j-1}^T r_{j-1} / (d_j^T Ad_j)$
- new guess $x_j = x_{j-1} + \alpha_j d_j$
- new residual $r_j = r_{j-1} - \alpha_j Ad_j$ with $z_j^T M z_i = 0$, $i \neq j$