MAT 421 Applied Computational Methods
Prof. Gardner (carl.gardner@asu.edu)

Reading: Section 1.7 (floating point) of Moler’s Numerical Computing with MATLAB.

Homework 1
Due: Fri Jan 13

Please label MAT 421 HW1, write up the problems neatly and in numerical order, box short answers, and write (2) omitted if problem (2) is not attempted. No need to restate problem.

(1) Verify that the three-point central difference formulas for $f'$ and $f''$ are second-order accurate. Include the constant appearing in front of $\Delta x^2$ in the error. (See Derivative Approximations and pages 4–5 of Numerical Methods for ODES & PDEs on the MAT 421 web page.)

(2) Verify that the one-sided difference formula

$$\frac{du}{dt} \approx \frac{u_{n+1} - u_n}{\Delta t}$$

is first-order accurate.

(3) Suppose a double precision floating point number is stored on a computer using 64 bits in the following way: sign 1 bit, exponent 8 bits, and mantissa 55 bits. A given real number $r$ is written as

$$r = \pm m2^n$$

where the mantissa $m$ satisfies $\frac{1}{2} \leq m < 1$ and $-128 \leq n \leq 127$. Give the following numbers in both base 2 and base 10 scientific notation:
(a) What is the largest positive number $\text{realmax}$ that can be stored?
(b) What is the smallest positive number $\text{realmin}$ that can be stored?
(c) What is the machine epsilon $\epsilon_M$ (without a phantom bit)?