Reading Assignment: Read Moler Sections 7.1–7.2, & 7.4 on ordinary differential equations. Keep reading the “UNIX Tutorial for Beginners” (Intro-Tutorial 4) & the “C Programming Notes” (Chapters 1–7).

Homework 3
Due: Fri Feb 12 (but keep until Mon)

(1) For \( du/dt = f(u) \), prove that the TR method is second-order accurate, using the definition of the local truncation error. Be sure to calculate the constant multiplying \( \Delta t^p \) in the global error.

(2) Prove that TR method is A-stable, but not L-stable. [Consider the model problem \( u' = au \) with \( a < 0 \).]

(3) Run the program BE_TR for 20 steps with \( dt = 0.2, 2., \) & 2.1. (Don’t turn in the output, but simply answer the questions & turn in the plot.)
   (a) What goes wrong at \( dt = 2. \)?
   (b) What do the results for \( dt = 2.1 \) illustrate?
   (c) Plot the exact & three numerical solutions for \( dt = 0.2 \), using Plot1.m.

Local Truncation Error

\[
(*) \quad \frac{du}{dt} = f(u), \quad u(t = 0) = u_0
\]

The discrete approximation is \( u_n \approx u(n\Delta t) \). The one-step discretized version of the initial value problem \( (*) \) can be written as

\[
u_{n+1} = u_n + \Delta t \, \Phi(u_n, \, u_{n+1}, \, \Delta t).
\]

We define the local truncation error (LTE = \( \Delta t \tau \)) for the initial value problem \( (*) \) by

\[
u((n + 1)\Delta t) = u(n\Delta t) + \Delta t \, \Phi(u(n\Delta t), \, u((n + 1)\Delta t), \, \Delta t) + \Delta t \tau
\]

where \( u \) is the exact solution of \( (*) \). A one-step method is \( p \)th order accurate if the (global) error \( \tau \sim \Delta t^p \).