(1) Solve the linear system $Ax = b$ in two steps, where

\[
A = \begin{bmatrix}
1 & 3 & 1 & 2 \\
2 & 6 & 4 & 8 \\
0 & 0 & 2 & 4
\end{bmatrix}, \quad b = \begin{bmatrix}
1 \\
3 \\
1
\end{bmatrix}
\]

First find a particular solution $x_p$ by transforming $[A \ b] \rightarrow \cdots \rightarrow [R \ d]$. Show all the steps. No credit will be given for just the answer.

$x_p =$ ________
(2) (a) Then find the special solutions $x_n$ by solving $Ax = 0 \rightarrow Rx = 0$.

(b) What is the general solution to $Ax = b$?
(3) (a) Find a basis for $C(A)$ and a basis for $N(A)$.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

(b) Find a basis for $C(A^T)$ and a basis for $N(A^T)$. 
(4) Least-squares fitting a line to data \( b = C + Dt = (4, 2, -1, 0, 0) \) at \( t = (-2, -1, 0, 1, 2) \): solve \( A^T A \hat{x} = A^T b \) where \( \hat{x} = (C, D) \) and

\[
A = \begin{bmatrix}
1 & t_1 \\
\vdots & \vdots \\
1 & t_m
\end{bmatrix}
\]

Line \( b = \) ________________
(5) Short answer questions:

(a) List all possible subspaces (there are four) of \( \mathbb{R}^3 \):

(b) \( Ax = b \) is solvable only if \( b \) is in which subspace of \( \mathbb{R}^m \)?

(c) The solutions to \( Ax = b \) form a subspace only if \[ \text{________} \]

(d) Suppose \( q_1, q_2, \ldots, q_n \) are a complete orthonormal basis for \( \mathbb{R}^n \). Set \( Q = [q_1 \ q_2 \ \ldots \ q_n] \). Then \( Q^TQ = \[ \text{________} \] and if \( b = a_1 q_1 + a_2 q_2 + \cdots + a_n q_n \), \[ a_2 = \[ \text{________} \]

In (e)–(j), suppose \( A = m \times n \) has \( r \) pivots.

(e) \( \dim\{ C(A) \} = \[ \text{________} \]

(f) \( \dim\{ C(A^T) \} = \[ \text{________} \]

(g) \( \dim\{ N(A) \} = \[ \text{________} \]

(h) \( \dim\{ N(A^T) \} = \[ \text{________} \]

(i) \[ \text{________} \] is the orthogonal complement to \( C(A^T) \) in \( \mathbb{R}^n \).

(j) \[ \text{________} \] is the orthogonal complement to \( C(A) \) in \( \mathbb{R}^m \).