Practice Test 1, Mathematics 275 (Prof. Gardner)

50 minutes, no books or notes or phones or ipods, calculator OK.

(1) Sketch the phase diagram & then sketch eight solution curves plus the equilibria indicating the behavior of solutions to

\[
\frac{dy}{dt} = y(1 - y)(y - 2)
\]

for various illustrative initial conditions.

(2) Solve \( \frac{dy}{dt} = y^2, \ y(1/2) = 2 \) using separation of variables.

(3) Using the integrating factor method, solve the initial value problem

\[
\frac{dy}{dt} + \frac{y}{t} = t^2, \quad y(1) = 2
\]

(4) Why is uniqueness not guaranteed for \( x \geq 0 \) for the IVP

\[
(\ast) \quad x \frac{dy}{dx} = 2y, \quad y(-1) = 1?
\]

Verify that the IVP (\( \ast \)) has infinitely many solutions of the form

\[
y(x) = \begin{cases} 
x^2 & x \leq 0 \\
\frac{c}{x^2} & x > 0
\end{cases}
\]

for any \( c \).