Helix  \[ \vec{r}(t) = (\cos t, \sin t, t) \]
\[ \vec{v}(t) = (-\sin t, \cos t, 1) \]
\[ \vec{a}(t) = (-\cos t, -\sin t, 0) \]
\[ |\vec{v}(t)| = \sqrt{2} = \frac{ds}{dt} \quad \text{speed} \]
\[ s = \sqrt{2} t \quad \text{distance (arc length)} \]
unit tangent vector \[ \vec{T} = \frac{\vec{v}}{|\vec{v}|} = \frac{\vec{v}}{\sqrt{2}} \]
\[ = \frac{1}{\sqrt{2}} (-\sin t, \cos t, 1) \]

Alternatively \[ \vec{r}(t) = (\cos t) \hat{i} + (\sin t) \hat{j} + t \hat{k} \]
\[ \vec{v}(t) = -(\sin t) \hat{i} + (\cos t) \hat{j} + \hat{k} \]
\[ \vec{a}(t) = -(\cos t) \hat{i} - (\sin t) \hat{j} \]

**Surfaces & Level Curves**

**Cone** \( z = f(x,y) = \sqrt{x^2 + y^2} \)

level curves \( f(x,y) = \sqrt{x^2 + y^2} = c \)

**circles**

**Paraboloid** \( z = f(x,y) = x^2 + y^2 \)

level curves \( f(x,y) = x^2 + y^2 = \text{const} \)

**circles**

**Table of Quadric Surfaces**

<table>
<thead>
<tr>
<th>Surface</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellipsoid</td>
<td>( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 )</td>
</tr>
<tr>
<td>Elliptic paraboloid</td>
<td>( \frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2} )</td>
</tr>
<tr>
<td>Hyperbolic paraboloid</td>
<td>( \frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2} )</td>
</tr>
<tr>
<td>Cone</td>
<td>( \frac{z^2}{a^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2} )</td>
</tr>
<tr>
<td>Hyperboloid (1 sheet)</td>
<td>( \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 )</td>
</tr>
<tr>
<td>Hyperboloid (2 sheets)</td>
<td>( \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1 )</td>
</tr>
</tbody>
</table>
Partial Derivatives

\[ f(x, y) = x^2 y^2 + xy + y \]

**Def**
\[
\frac{\partial f(x, y)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}
\]
\[
\frac{\partial f(x, y)}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}
\]

\[ \frac{\partial f}{\partial x} = 2xy^2 + y \quad \frac{\partial f}{\partial y} = 2x^2y + x + 1 \]

**Ex** Cone \( f(x, y) = \sqrt{x^2 + y^2} \)

\[ \frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} \quad \frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2 + y^2}} \]

At \((0, 2)\), \( \frac{\partial f}{\partial x}(0, 2) = 0 \) \& \( \frac{\partial f}{\partial y}(0, 2) = 1 \)

Note that \( \frac{\partial f}{\partial x} \) \& \( \frac{\partial f}{\partial y} \) are undefined at \((0, 0)\)

**Paraboloid** \( f(x, y) = x^2 + y^2 \)

\[ \frac{\partial f}{\partial x} = 2x \quad \frac{\partial f}{\partial y} = 2y \]

\[ \frac{\partial f}{\partial x}(0, 2) = 0 \quad \frac{\partial f}{\partial y}(0, 2) = 4 \]

**Hyperbolic paraboloid** (saddle)
\[ f(x, y) = y^2 - x^2 \]

To graph: \( f(x, 0) = -x^2 \)
\[ f(0, y) = y^2 \]

level curves \( z = \text{const} = y^2 - x^2 \)
saddle point at \((0, 0)\)

\[ \frac{\partial f}{\partial x} = -2x, \quad \frac{\partial f}{\partial x}(0, 0) = 0, \quad \frac{\partial f}{\partial y} = 2y, \quad \frac{\partial f}{\partial y}(0, 0) = 0 \]

But
\[ \frac{\partial^2 f}{\partial x^2}(0, 0) = -2, \quad \frac{\partial^2 f}{\partial y^2}(0, 0) = 2 \]

(Here \( \frac{\partial^2 f}{\partial x \partial y} = 0 \))
partial differential equations (PDEs)

1D heat equation

\[ \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad u(x,t) = T(x,t) - T_{\text{amb}} \]

shorthand notation \[ u_t = u_{xx} \]

one solution (the fundamental solution) is

\[ u(x,t) = \frac{1}{\sqrt{4\pi t}} e^{-\frac{x^2}{4t}} \]

\[ \frac{\partial u}{\partial t} = -\frac{1}{2t^{3/2}} e^{-\frac{x^2}{4t}} + \frac{1}{\sqrt{t}} e^{-\frac{x^2}{4t}} \left( \frac{x^2}{4t^2} \right) \]

\[ \frac{\partial u}{\partial x} = -\frac{2x}{4t} \frac{1}{\sqrt{t}} e^{-\frac{x^2}{4t}} = -\frac{x}{2t^{3/2}} e^{-\frac{x^2}{4t}} \]

\[ \frac{\partial^2 u}{\partial x^2} = -\frac{1}{2t^{3/2}} e^{-\frac{x^2}{4t}} + \frac{x^2}{4t^{5/2}} e^{-\frac{x^2}{4t}} = \frac{\partial u}{\partial t} \checkmark \]

1D first-order wave equation

\[ \frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0 \quad , \quad u(\infty,t) = U_0(x) \equiv \text{Asin(}kx\text{)} \]

solution is \[ u(x,t) = U_0(x-ct) \equiv \text{Asin(}k(x-ct)\text{)} \]

\[ \frac{\partial u}{\partial t} = -kcA \cos(k(x-ct)) \]

\[ \frac{\partial u}{\partial x} = kA \cos(k(x-ct)) \text{, so } u_t + cu_x = 0 \checkmark \]

traveling wave \[ U_0(x-ct) \]

\[ x = x_0 + ct \text{ or } x_0 = x - ct \text{, so } U_0(x-ct) = U_0(x_0) \]