Have you ever been camping in the rain? If not, try to imagine it. You are sitting in a tent with the rain flaps down, with nothing to do—the weather report was wrong and you wish that you had picked another weekend for this experience. As we all know, predicting weather is not an exact science. The weather is an example of a random phenomenon. Random phenomena are occurrences that vary from day-to-day and case-to-case. In addition to the weather, rolling dice in Monopoly, drilling for oil, and driving your car are all examples of random phenomena.

Although we never know exactly how a random phenomenon will turn out, we can often calculate a number called a probability that it will occur in a certain way. We will now begin to introduce some basic probability terminology.

Sample Spaces and Events

Our first step in calculating the probability of a random phenomenon is to determine the sample space of an experiment.

**Definitions**

An experiment is any observation of a random phenomenon. The different possible results of the experiment are called outcomes. The set of all possible outcomes for an experiment is called a sample space.

If we observe the results of flipping a single coin, we have an example of an experiment. In the Oscar-winning film No Country for Old Men, Anton flips a coin to bet on a person’s life. The possible outcomes are head and tail, so a sample space for the experiment would be the set {head, tail}.

**Example 1**

Finding Sample Spaces

Determine a sample space for each experiment.

a) We select an iPhone from a production line and determine whether it is defective.
b) Three children are born to a family and we note the birth order with respect to gender.
c) We select one card from a standard 52-card deck,* and then without returning the card to the deck, we select a second card. We will assume the order in which we select the cards is important.
d) We roll two dice and observe the pair of numbers showing on the top faces.

*See Figure 13.1 in Chapter 13 for a picture of a standard 52-card deck.
**SOLUTION:** In each case, we find the sample space by collecting the outcomes of the experiment into a set.

a) This sample space is \{defective, nondefective\}.

b) In this experiment, we want to know not only how many boys and girls are born but also the birth order. For example, a boy followed by two girls is not the same as two girls followed by a boy. The tree diagram* in Figure 14.1 helps us find the sample space.

There are two ways that the first child can be born, followed by two ways for the second child, and finally two ways for the third. If we abbreviate “boy” by “b” and “girl” by “g,” following the branches of the tree diagram gives us the following sample space:

\[ \{bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg\} \]

*We introduced tree diagrams in Chapter 13.

**KEY POINT**

An event is a subset of a sample space.

*DEFINITION* In probability theory, an *event* is a subset of the sample space.
Keep in mind that any subset of the sample space is an event, including such extreme subsets as the empty set, one-element sets, and the whole sample space.

**Some Good Advice**

Although we usually describe events verbally, you should remember that an event is always a subset of the sample space. You can use the verbal description to identify the set of outcomes that make up the event.

Example 2 illustrates some events from the sample spaces in Example 1.

**EXAMPLE 2** Describing Events as Subsets

Write each event as a subset of the sample space.

a) A head occurs when we flip a single coin.
b) Two girls and one boy are born to a family.
c) A total of five occurs when we roll a pair of dice.

**SOLUTION:**

a) The set \{head\} is the event.
b) Noting that the boy can be the first, second, or third child, the event is \{ bgg, gbg, ggb \}.
c) The following set shows how we can roll a total of five on two dice: \{(1, 4), (2, 3), (3, 2), (4, 1)\}.

Now try Exercises 7 to 14.

We will use the notions of outcome, sample space, and event to compute probabilities. Intuitively, you may expect that when rolling a fair die, each number has the same chance, namely \(\frac{1}{6}\), of showing. In predicting weather, a forecaster may state that there is a 30% chance of rain tomorrow. You may believe that you have a 50–50 chance (that is, a 50% chance) of getting a job offer in your field. In each of these examples, we have assigned a number between 0 and 1 to represent the likelihood that the outcome will occur. You can interpret probability intuitively as in the diagram below at left.

**Definitions** The probability of an outcome in a sample space is a number between 0 and 1 inclusive. The sum of the probabilities of all the outcomes in the sample space must be 1. The probability of an event \(E\), written \(P(E)\), is defined as the sum of the probabilities of the outcomes that make up \(E\).

One way to determine probabilities is to use empirical information. That is, we make observations and assign probabilities based on those observations.

**Empirical Assignment of Probabilities** If \(E\) is an event and we perform an experiment several times, then we estimate the probability of \(E\) as follows:

\[ P(E) = \frac{\text{the number of times } E \text{ occurs}}{\text{the number of times the experiment is performed}}. \]

This ratio is sometimes called the relative frequency of \(E\).

---

*Quiz Yourself answers begin on page 778.

1. Do not confuse \(P(E)\), which is the notation for the probability of an event, with \(P(n, r)\), which is the notation for the number of permutations of \(n\) objects taken \(r\) at a time (see Section 13.3).
EXAMPLE 3 Using Empirical Information to Assign Probabilities

A pharmaceutical company is testing a new flu vaccine. The experiment is to inject a patient with the vaccine and observe the occurrence of side effects. Assume that we perform this experiment 100 times and obtain the information in Table 14.1.

Based on Table 14.1, if a physician injects a patient with this vaccine, what is the probability that the patient will develop severe side effects?

**SOLUTION:** In this case, we base our probability assignment of the event that severe side effects occur on observations. We use the formula for the relative frequency of an event as follows:

\[ P(\text{severe side effects}) = \frac{\text{the number of times severe side effects occurred}}{\text{the number of times the experiment was performed}} = \frac{3}{100} = 0.03 \]

Thus, from this empirical information, we can expect a 3% chance that there will be severe side effects from this vaccine.

Now try Exercises 29 to 32.

EXAMPLE 4 Investigating Marital Data

Table 14.2* summarizes the marital status of men and women in the United States in 2006. All numbers represent number of thousands. If we randomly pick a male, what is the probability that he is divorced?

**SOLUTION:** It is important to recognize that not all entries in Table 14.2 are relevant to the question that you were asked. We are only interested in males, so the line that we have highlighted in the table contains all the information that we need to solve the problem. Therefore, we consider our sample space to be the

\[ 60,955 + 2,908 + 10,818 + 2,210 + 39,435 = 116,326 \]

males.

The event, call it \( D \), is the set of 10,818 men who are divorced. Therefore, the probability that we would select a divorced male is

\[ P(D) = \frac{n(D)}{n(S)} = \frac{10,818}{116,326} \approx 0.093. \]

Now try Exercises 41, 42, and 47 to 50.

KEY POINT

We can use counting formulas to compute probabilities.

---

**TABLE 14.1** Summary of side effects of the flu vaccine.

<table>
<thead>
<tr>
<th>Side Effects</th>
<th>Number of Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>72</td>
</tr>
<tr>
<td>Mild</td>
<td>25</td>
</tr>
<tr>
<td>Severe</td>
<td>3</td>
</tr>
</tbody>
</table>

**Quiz Yourself 2**

Use Table 14.1 to find the probability that a patient receiving the flu vaccine will experience no side effects.

**Quiz Yourself 3**

Using Table 14.2, if we select a widowed person, what is the probability that the person is a woman?

---

*U.S. Bureau of the Census.
In order to understand the difference between using theoretical and empirical information, compare these two experiments:

Experiment one—Without looking, draw a ball from a box, note its color, and return the ball to the box. If you repeat this experiment 100 times and get 60 red balls and 40 blue balls, based on this empirical information, you would expect that the probability of getting a red ball on your next draw to be \( \frac{60}{100} = 0.60 \).

Experiment two—Draw a 5-card hand from a standard 52-card deck. As you will soon see, you can use theoretical information, namely, combination formulas from Chapter 13, to calculate the probability that all cards will be hearts.

**EXAMPLE 5 Using Counting Formulas to Calculate Probabilities**

Assign probabilities to the outcomes in the following sample spaces.

a) We flip three fair coins. What is the probability of each outcome in this sample space?

b) We draw a 5-card hand randomly from a standard 52-card deck. What is the probability that we draw one particular hand?

**SOLUTION:**

a) This sample space has eight outcomes, as we show in Figure 14.2. Because the coins are fair, we expect that heads and tails are equally likely to occur. Therefore, it is reasonable to assign a probability of \( \frac{1}{8} \) to each outcome in this sample space.

![Figure 14.2](image)

**FIGURE 14.2** Eight theoretically possible outcomes for flipping three coins.

b) In Chapter 13, we found that there are \( \binom{52}{5} = 2,598,960 \) different ways to choose 5 cards from a deck of 52. Because we are drawing the cards randomly, each hand has the same chance of being drawn. Therefore, the probability of drawing any one hand is \( \frac{1}{2,598,960} \).

In a sample space with equally likely outcomes, it is easy to calculate the probability of any event by using the following formula.

**Calculating Probability When Outcomes Are Equally Likely** If \( E \) is an event in a sample space \( S \) with all equally likely outcomes, then the probability of \( E \) is given by the formula:

\[
P(E) = \frac{n(E)}{n(S)}
\]

*Recall that \( n(E) \) is the cardinal number of set \( E \) (see Section 2.1).
Math in Your Life*

Why Does It Always Happen to You?

When you wait at a toll booth, why does it seem that the line of cars next to you moves faster than your line? If you look into a sock drawer, do you notice how many unmatched socks there are? Why does the buttered side of a slice of bread almost always land face down if you drop the bread while making a sandwich? Do such annoyances affect only you? Or is there a mathematical explanation?

First let’s consider the socks. Suppose you have 10 pairs of socks in a drawer and lose 1 sock, destroying a pair. Of the 19 remaining socks, there is only 1 unpaired sock. Therefore, if you lose a second sock, the probability of losing a paired sock is \( \frac{18}{19} \). Now you have 2 unpaired socks and 16 paired socks. The probability that the third sock you lose will be part of a pair is still overwhelming. Continuing this line of thought, you see that it will be quite a while before probability theory predicts that you can expect to lose an unpaired sock.

The problem with the buttered bread is simple to explain. In fact you might conduct an experiment to simulate this situation with an object such as a computer mouse pad. If you slide the object off the edge of a table, as it begins to fall it rotates halfway around so that the top side is facing down. However, there is usually not enough time for the object to rotate back to the upward position before it hits the floor. You could conduct this experiment 100 times and determine the empirical probability that an object you slide off the table will land face down. So again, it’s not bad luck—it’s probability.

The slow-moving line at the toll booth is the easiest to explain. If we assume that delays in any line occur randomly, then one of the lines—yours, the one to the left, or the one to the right—will move fastest. Therefore, the probability that the fastest line will be yours is only \( \frac{1}{3} \).

EXAMPLE 6 Computing Probability of Events

a) What is the probability in a family with three children that two of the children are girls?

b) What is the probability that a total of four shows when we roll two fair dice?

c) If we draw a 5-card hand from a standard 52-card deck, what is the probability that all 5 cards are hearts?

d) Harold, Kumar, Neil, and Vanessa are friends who belong to their college’s 10-person international relations club. Two people from the club will be selected randomly to attend a conference at the United Nations building. What is the probability that two of these four friends will be selected?

**SOLUTION:** In each situation, we assume that the outcomes are equally likely.

a) We saw in Example 1 that there are eight outcomes in this sample space. We denote the event that two of the children are girls by the set \( G = \{bgg, gbg, ggb\} \). Thus,

\[
P(G) = \frac{n(G)}{n(S)} = \frac{3}{8}.
\]

b) The sample space for rolling two dice has 36 ordered pairs of numbers. We will represent the event “rolling a four” by \( F \). Then \( F = \{(1, 3), (2, 2), (3, 1)\} \). Thus,

\[
P(F) = \frac{n(F)}{n(S)} = \frac{3}{36} = \frac{1}{12}.
\]

c) From Example 5, we know that there are \( C(52, 5) \) ways to select a 5-card hand from a 52-card deck. If we want to draw only hearts, then we are selecting 5 hearts from the...

---

*This Math in Your Life is based on Robert Matthews, “Murphy’s Law or Coincidence.” Reader’s Digest, March 1998, pp. 25–30.

†Questions c) and d) require counting formulas from Chapter 13.
The Basics of Probability Theory

13 available, which can be done in \(C(13, 5)\) ways. Thus, the probability of selecting all five cards to be hearts is

\[
\frac{C(13, 5)}{C(52, 5)} = \frac{1,287}{2,598,960} \approx 0.000495.
\]

d) The sample space, \(S\), consists of all the ways we can select two people from the 10 members in the club. As you know from Chapter 13, we can choose 2 people from 10 in \(C(10, 2) = \frac{10!}{8!\cdot 2!} = \frac{10 \cdot 9}{2} = 45\) ways. The event, call it \(E\), consists of the ways we can choose two of the four friends. This can be done in \(C(4, 2) = 6\) ways. Because all elements in \(S\) (choices of two people) are equally likely, the probability of \(E\) is

\[
P(E) = \frac{n(E)}{n(S)} = \frac{C(4, 2)}{C(10, 2)} = \frac{6}{45} = \frac{2}{15}.
\]

Now try Exercises 15 to 20.

### Some Good Advice

If the outcomes in a sample space are not equally likely, then you cannot use the formula

\[
P(E) = \frac{n(E)}{n(S)}.
\]

In that case, to find \(P(E)\), you must add the probabilities of all the individual outcomes in \(E\).

Suppose that we have a sample space with equally likely outcomes. An event, \(E\), can contain none, some, or all of the outcomes in the sample space \(S\), so we can say

\[0 \leq n(E) \leq n(S)\]

Dividing this inequality by the positive quantity \(n(S)\), we get the inequality

\[
0 \leq \frac{n(E)}{n(S)} \leq \frac{n(S)}{n(S)}
\]

which simplifies to \(0 \leq \frac{n(E)}{n(S)} \leq 1\). This gives us the first probability property listed below. The other properties are easy to see.

### Basic Properties of Probability

Assume that \(S\) is a sample space for some experiment and \(E\) is an event in \(S\).

1. \(0 \leq P(E) \leq 1\)
2. \(P(\emptyset) = 0\)
3. \(P(S) = 1\)

### Probability and Genetics

In the nineteenth century, the Austrian monk Gregor Mendel noticed while cross-breeding plants that often a characteristic of the plants would disappear in the first-generation offspring but reappear in the second generation. He theorized that the first-generation plants contained a hidden factor (which we now call a gene) that was somehow transmitted to the second generation to enable the characteristic to reappear.

To check his theory, he selected a characteristic such as seed color—some peas had yellow seeds and some had green. Then when he was sure that he had bred plants that would produce only yellow seeds or green seeds, he was ready to begin his experiment. Mendel believed that one of the colors was dominant and the other was recessive. Which turned out to be the case, because when yellow-seeded plants were crossed with green-seeded plants, the offspring had yellow seeds. When Mendel crossed these offspring for a second generation, he found that 6,022 plants had yellow seeds and 2,001 had green seeds.
which is almost exactly a ratio of 3 to 1. Because Mendel was skilled in mathematics as well as biology, he gave the following explanation for what he had observed.

We will represent the gene that produces the yellow seed by \( Y \) and the gene that produces the green seed by \( g \). The uppercase \( Y \) indicates that yellow is dominant and the lowercase \( g \) indicates that green is recessive.*

Figure 14.3 shows the possible genetic makeup of the offspring from crossing a plant with pure yellow seeds and a plant with pure green seeds. Every one of these offspring has a \( Yg \) pair of genes. Because “yellow seed” is dominant over “green seed,” every plant in the first generation will have yellow seeds.

![Figure 14.3](image)

Each offspring will have one \( Y \) gene from parent 1 and one \( g \) gene from parent 2, and each will have yellow seeds.

**Figure 14.3** The possible first-generation offspring we obtain by crossing pure yellow and pure green parents.

Figure 14.4 shows the possible genetic outcomes that can occur if we cross two first-generation pea plants. We summarize Figure 14.4 in Table 14.4, which is called a **Punnett square**.

![Figure 14.4](image)

Each offspring will have either a \( Y \) gene or a \( g \) gene from each parent. Only the seeds of the offspring with the \( gg \) pair of genes will be green.

**Figure 14.4** The possible second-generation offspring we obtain by crossing the first-generation offspring of pure yellow and pure green parents.

<table>
<thead>
<tr>
<th>First-Generation Plant</th>
<th>First-Generation Plant</th>
<th>Second Generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y )</td>
<td>( g )</td>
<td>( YY )</td>
</tr>
<tr>
<td>( gY )</td>
<td>( gg )</td>
<td></td>
</tr>
</tbody>
</table>

**Table 14.3** The genetic possibilities of crossing two plants that each have one yellow-seed and one green-seed gene.

As you can see in Table 14.3, there are four things that can happen when we cross two first-generation plants. We can get a \( YY \), \( Yg \), \( gY \), or \( gg \) type of plant. Of these four possibilities, only the \( gg \) will result in green seeds, which explains why Mendel saw the recessive characteristic return in roughly one fourth of the second-generation plants.

---

*Biology books customarily use slightly different notation to indicate genes. For example, to indicate that yellow is dominant over green, you may see the notation \( Yy \). The capital \( Y \) represents the yellow dominant gene and the lowercase \( y \) indicates the recessive green gene.
EXAMPLE 7 Using Probability to Explain Genetic Diseases

Sickle-cell anemia is a serious inherited disease that is about 30 times more likely to occur in African American babies than in non–African American babies. A person with two sickle-cell genes will have the disease, but a person with only one sickle-cell gene will be a carrier of the disease.

If two parents who are carriers of sickle-cell anemia have a child, what is the probability of each of the following:

a) The child has sickle-cell anemia?

b) The child is a carrier?

c) The child is disease free?

SOLUTION: Table 14.4 shows the genetic possibilities when two people who are carriers of sickle-cell anemia have a child. We will denote the sickle-cell gene by \( s \) and the normal gene by \( n \). We use lowercase letters to indicate that neither \( s \) nor \( n \) is dominant.

From Table 14.4, we see that there are four equally likely outcomes for the child.

- The child receives two sickle-cell genes and therefore has the disease.
- The child receives a sickle-cell gene from the first parent and a normal gene from the second parent and therefore is a carrier.
- The child receives a normal gene from the first parent and a sickle-cell gene from the second parent and therefore is a carrier.
- The child receives two normal genes and therefore is disease-free.

From this analysis, it is clear that

\[
\begin{align*}
\text{a)} & \quad P(\text{the child has sickle-cell anemia}) = \frac{1}{4} \\
\text{b)} & \quad P(\text{the child is a carrier}) = \frac{1}{2} \\
\text{c)} & \quad P(\text{the child is normal}) = \frac{1}{4} 
\end{align*}
\]

Now try Exercises 33 to 40.

**KEY POINT** In computing odds remember “against” versus “for.”

**Odds**

We often use the word odds to express the notion of probability. When we do this, we usually state the odds against something happening. For example, before the 2008 Kentucky Derby, Las Vegas oddsmakers had set the odds against Big Brown, the eventual winner of the derby, to be 4 to 1.

When you calculate odds against an event, it is helpful to think of what is against the event as compared with what is in favor of the event. For example, if you roll two dice and want to find the odds against rolling a seven, you think that there are 30 pairs that will give you a nonseven and six that will give you a seven. Therefore, the odds against rolling a total of seven are 30 to 6. We write this as 30:6. Just like reducing a fraction, we can write these odds as 5:1. Sometimes you might see odds written as a fraction, such as 30/6, but, for the most part, we will avoid using this notation.

**DEFINITION** If the outcomes of a sample space are equally likely, then the odds against an event \( E \) are simply the number of outcomes that are against \( E \) compared with the number of outcomes in favor of \( E \) occurring. We would write these odds as \( n(E') : n(E) \), where \( E' \) is the complement of event \( E \).

---

\*We discussed the complement of a set in Section 2.3.
Recall that in Example 1, you saw there were eight ways for boys and girls to be born in order in a family. To find the odds against all children being of the same gender, you think of the six outcomes that are against this happening versus the two outcomes that are in favor, as in the accompanying diagram. Therefore, the odds against all three children being of the same gender are 6:2, which we can reduce and rewrite as 3:1. We could also say that the odds in favor of all children being of the same gender are 1:3. It is important to understand that the odds in favor of an event are not the same as the probability of an event. In the example we are discussing, the probability of all children being the same gender is \( \frac{2}{8} = \frac{1}{4} \).

**PROBLEM SOLVING**

**The Analogies Principle**

Making an analogy with a real-life situation helps you to remember the meaning of mathematical terminology. For example, when General Custer fought Chief Sitting Bull at the battle of Little Big Horn, Custer had about 650 soldiers and Sitting Bull had 2,500 braves. If we think of how many were for Custer and how many were against him, we might say that the odds against Custer winning were 2,500 to 650. Similarly, the odds against Sitting Bull winning were 650 to 2,500.

**EXAMPLE 8  Calculating Odds on a Roulette Wheel**

A common type of roulette wheel has 38 equal-size compartments. Thirty-six of the compartments are numbered 1 to 36 with half of them colored red and the other half black. The remaining 2 compartments are green and numbered 0 and 00. A small ball is placed on the spinning wheel and when the wheel stops, the ball rests in one of the compartments. What are the odds against the ball landing on red?

**SOLUTION:** This is an experiment with 38 equally likely outcomes. Because 18 of these are in favor of the event “the ball lands on red” and 20 are against the event, the odds against red are 20 to 18. We can write this as 20:18, which we may reduce to 10:9.

Although we have defined odds in terms of counting, we also can think of odds in terms of probability. Notice in the next definition we are comparing “probability against” with “probability for.”
PROBABILITY FORMULA FOR COMPUTING ODDS  If \( E' \) is the complement of the event \( E \), then the odds against \( E \) are

\[
\frac{P(E')}{P(E)}
\]

You may have been surprised that we have not used the “\( a:b \)” notation in this definition of odds. However, we have a good reason for doing this. If the probability of an event \( E \) is 0.30, then it is awkward to say that the odds against \( E \) are 0.70 to 0.30. It is better to say

\[
\text{odds against } E = \frac{\text{probability of } E'}{\text{probability of } E} = \frac{0.70}{0.30} = \frac{70}{30} = 70:30 = 7:3.
\]

We could then say that the odds against \( E \) are 70 to 30, or 7 to 3. We will use this approach in Example 9.

**EXAMPLE 9  Using the Probability Formula for Computing the Odds in a Football Game**

Suppose that the probability of Green Bay winning the Super Bowl is 0.35. What are the odds against Green Bay winning the Super Bowl?

**SOLUTION:** Recall from Section 1.1 on problem solving that a diagram often helps us remember what to do. Call the event “Green Bay wins the Super Bowl” \( G \). We illustrate this in Figure 14.5.

Thus, the odds against Green Bay winning the Super Bowl are

\[
\frac{P(G')}{P(G)} = \frac{0.65}{0.35} = \frac{65 \times 100}{35} = 13 : 7.
\]

In this case, we would say the odds against Green Bay winning the Super Bowl are 13 to 7.

Now try Exercises 21 to 24. *

Now that we have introduced some of the basic terminology and properties of probability, in the next section we will learn rules for calculating probabilities.

---

**Exercises 14.1**

**Looking Back**

*These exercises follow the general outline of the topics presented in this section and will give you a good overview of the material that you have just studied.*

1. In Example 1(b), what method did we use to determine the sample space?
2. Although we usually describe events verbally, you should remember that an event is always what?
3. In Example 5(b), how did we find the number of objects in the sample space?
4. In Example 9, we knew that the probability of Green Bay winning the Super Bowl was 0.35. How did we find the odds against Green Bay winning?

---

5. In the Math in Your Life box, what was our explanation as to why the buttered side of a falling piece of bread lands face down? Why is it reasonable that your line at a toll booth would not be the fastest?
6. What was the point of our discussion about Custer and Sitting Bull?

**Sharpening Your Skills**

*In Exercises 7–10, write each event as a set of outcomes. If the event is large, you may describe the event without writing it out.*

7. When we roll two dice, the total showing is seven.
8. When we roll two dice, the total showing is five.
9. We flip three coins and obtain more (\( h \))eads than (\( t \))ails.
10. We select a red face card from a standard 52-card deck.

*Before doing these exercises, you may find it useful to review the note How to Succeed at Mathematics on page xix.
In Exercises 11–14, use the given spinner to write the event as a set of outcomes. Abbreviate “red” as “r,” “blue” as “b,” and “yellow” as “y.”

11. Red appears exactly once when we spin the given spinner two times.
12. Yellow appears at least once when we spin the given spinner two times.
13. Blue appears exactly twice in three spins of the spinner.

15. We are rolling two four-sided dice having the numbers 1, 2, 3, and 4 on their faces. Outcomes in the sample space are pairs such as (1, 3) and (4, 4).
   a. How many elements are in the sample space?
   b. Express the event “the total showing is even” as a set.
   c. What is the probability that the total showing is even?
   d. What is the probability that the total showing is greater than six?

16. You select three digital picture frames from a production line to determine if they are defective or nondefective. Outcomes in the sample space are represented by strings such as dnd and nnm.
   a. How many elements are in the sample space?
   b. Express the event “exactly one frame is defective” as a set.
   c. What is the probability that exactly one frame is defective?
   d. What is the probability that more frames are defective than nondefective?

17. Singers (C)arrie, (M)ariah, (K)eith, and (J)ustin are to perform in a talent contest and the order of their appearance will be chosen randomly. Outcomes in the sample space are represented by strings of letters such as CKMJ and JKCM.
   a. How many elements are in the sample space?
   b. Express the event “the women (C and M) do not perform consecutively” as a set.
   c. What is the probability that the women do not perform consecutively?
   d. What is the probability that Carrie will perform last?

18. We are flipping four coins. Outcomes in the sample space are represented by strings of Hs and Ts such as TTHT and HHTT.
   a. How many elements are in this sample space?
   b. Express the event “there are more heads than tails” as a set.
   c. What is the probability that there are more heads than tails?
   d. What is the probability that there are an equal number of heads and tails?

19. An experimenter testing for extrasensory perception has five cards with pictures of a (s)tar, a (c)ircle, (w)iggly lines, a (d)ollar sign, and a (h)eart. She selects two cards without replacement. Outcomes in the sample space are represented by pairs such as (s, d) and (h, c).
   a. How many elements are in this sample space?
   b. Express the event “a star appears on one of the cards” as a set.
   c. What is the probability that a star appears on one of the cards?
   d. What is the probability that a heart does not appear?

20. You have bought stock in Apple and Dell and each stock can either (i)ncrease in value, (d)ecrease in value, or (s)tay the same. Outcomes in the sample space are represented by pairs such as (i, d) and (d, d).
   a. How many elements are in this sample space?
   b. Express the event “Apple increases” as a set.
   c. What is the probability that Apple increases?
   d. What is the probability that Dell does not stay the same?

In Exercises 21–24, a) Find the probability of the given event. b) Find the odds against the given event.

21. A total of nine shows when we roll two fair dice.
   a. What is the probability that the total showing is greater than six?
   b. What is the probability that the total showing is even?

22. A total of three shows when we roll two fair dice.
   a. What is the probability that there are an equal number of heads and tails?
   b. What is the probability that more frames are defective than nondefective?

23. We draw a heart when we select 1 card randomly from a standard 52-card deck.
   a. What is the probability that there are more heads than tails?
   b. What is the probability that all cards are of the same suit?
   c. What is the probability that all cards are red?
   d. What is the probability that the women do not perform consecutively?

24. We draw a face card when we select 1 card randomly from a standard 52-card deck.
   a. What is the probability that more frames are defective than nondefective?
   b. What is the probability that all cards are diamonds?
   c. What is the probability that all cards are face cards?
   d. What is the probability that all cards are of the same suit?

25. The residents of a small town and the surrounding area are divided over the proposed construction of a sprint car racetrack in the town. Use the following table to answer Exercises 29 and 30.

<table>
<thead>
<tr>
<th></th>
<th>Support Racetrack</th>
<th>Oppose Racetrack</th>
</tr>
</thead>
<tbody>
<tr>
<td>Live in Town</td>
<td>1,512</td>
<td>2,268</td>
</tr>
<tr>
<td>Live in Surrounding Area</td>
<td>3,528</td>
<td>1,764</td>
</tr>
</tbody>
</table>

26. If a newspaper reporter randomly selects a person to interview from these people,
   a. what is the probability that the person supports the racetrack?
   b. what are the odds in favor of the person supporting the racetrack?

27. If a newspaper reporter randomly selects a person from town to interview,
   a. what is the probability that the person supports the racetrack?
   b. what are the odds in favor of the person supporting the racetrack?

Applying What You’ve Learned

29. In a given year, 2,048,861 males and 1,951,379 females were born in the United States. If a child is selected randomly from this group, what is the probability that it is a female?

30. Recently, the FBI reported that there were 7,160 hate crimes in the United States. Of these, 3,919 were based on race, 1,227 on religion, 1,017 on sexual orientation, 944 on ethnicity, and 53 on disability. (Assume that no crime was reported in two
categories.) If you selected one of these crimes randomly, what is the probability that it would not be based on race?

33. The following table lists some of the empirical results that Mendel obtained in his experiments in cross-breeding pea plants.

<table>
<thead>
<tr>
<th>Characteristics That Were Crossbred</th>
<th>First-Generation Plants</th>
<th>Second-Generation Plants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tall vs. short</td>
<td>All tall</td>
<td>787 tall 277 short</td>
</tr>
<tr>
<td>Smooth seeds vs. wrinkled seeds</td>
<td>All smooth seeds</td>
<td>5,474 smooth 1,850 wrinkled</td>
</tr>
</tbody>
</table>

Assume that we are cross-breeding genetically pure tall plants with genetically pure short plants. Use this information to assign the probability that a second-generation plant will be short. How consistent is this with the theoretical results that Mendel derived?

34. Assume that we are cross-breeding genetically pure smooth-seed plants with genetically pure wrinkled-seed plants. Use the information provided in Exercise 33 to assign the probability that a second-generation plant will have smooth seeds. How consistent is this with the theoretical results that Mendel derived?

In Exercises 35 and 36, construct a Punnett square as we did in Example 7 to describe the genetic possibilities for a child whose two parents are carriers of cystic fibrosis.

35. One parent who has sickle-cell anemia and one parent who is a carrier have a child. Find the probability that the child is a carrier of sickle-cell anemia.

36. One parent who has sickle-cell anemia and one parent who is a carrier have a child. Find the probability that the child has sickle-cell anemia.

In cross-breeding snapdragons, Mendel found that flower color does not dominate, as happens with peas. For example, a snapdragon with one red and one white gene will have pink flowers. In Exercises 37 and 38, analyze the cross-breeding experiment as we did in the discussion prior to Example 7.

37. a. Construct a Punnett square showing the results of crossing a purebred white snapdragon with a purebred red one.
   b. What is the probability of getting red flowers in the first-generation plants? What is the probability of getting white? Of getting pink?

38. a. If we cross two pink snapdragons, draw a Punnett square that shows the results of crossing two of these first-generation plants.
   b. What is the probability of getting red flowers in the second-generation plants? What about white? Pink?

39. Cystic fibrosis is a serious inherited lung disorder that often causes death in victims during early childhood. Because the gene for this disease is recessive, two apparently healthy adults, called carriers, can have a child with the disease. We will denote the normal gene by N and the cystic fibrosis gene by c to indicate its recessive nature.

   a. Construct a Punnett square as we did in Example 7 to describe the genetic possibilities for a child whose two parents are carriers of cystic fibrosis.
   b. What is the probability that this child will have the disease?

40. From the Punnett square in Exercise 39, what is the probability that a child of two carriers will
   a. be normal?
   b. be a carrier?

41. Assume that the following table summarizes a survey involving the relationship between living arrangements and grade point average for a group of students.

<table>
<thead>
<tr>
<th></th>
<th>On Campus</th>
<th>At Home</th>
<th>Apartment</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 2.5</td>
<td>98</td>
<td>40</td>
<td>44</td>
<td>182</td>
</tr>
<tr>
<td>2.5 to 3.5</td>
<td>64</td>
<td>25</td>
<td>20</td>
<td>109</td>
</tr>
<tr>
<td>Over 3.5</td>
<td>17</td>
<td>4</td>
<td>8</td>
<td>29</td>
</tr>
<tr>
<td>Totals</td>
<td>179</td>
<td>69</td>
<td>72</td>
<td>320</td>
</tr>
</tbody>
</table>

If we select a student randomly from this group, what is the probability that the student has a grade point average of at least 2.5?

42. Using the data in Exercise 41, if a student is selected randomly, what is the probability that the student lives off campus?

Use this replica of the Monopoly game board to answer Exercises 43–46.
43. Assume that your game piece is on the Electric Company. If you land on either St. James Place, Tennessee Avenue, or New York Avenue, you will go bankrupt. What is the probability that you avoid these properties?

44. Assume that your game piece is on Pacific Avenue. If you land on either Park Place or Boardwalk, you will go bankrupt. What is the probability that you avoid these properties?

45. Your game piece is on Virginia Avenue. What is the probability that you will land on a railroad on your next move?

46. Your game piece is on Pennsylvania Avenue. What is the probability that you will have to pay a tax on your next move?

In Exercises 47–50 assume that we are randomly picking a person described in Table 14.2 from Example 4.

47. If we pick a divorced person, what is the probability that the person is a woman?

48. If we pick a woman, what is the probability that the person is married but not separated?

49. If we pick a never-married person, what is the probability that the person is a man?

50. If we pick a man, what is the probability that the person is a widower?

In horse racing, a trifecta is a race in which you must pick the first-, second-, and third-place winners in their proper order to win a payoff.

51. If eight horses are racing and you randomly select three as your bet in the trifecta, what is the probability that you will win? (Assume that all horses have the same chance to win.)

52. If 10 horses are racing and you randomly select 3 as your bet in the trifecta, what is the probability that you will win? (Assume that all horses have the same chance to win.)

53. If the odds against event $E$ are 5 to 2, what is the probability of $E$?

54. If $P(E) = 0.45$, then what are the odds against $E$?

55. If the odds against the U.S. women’s soccer team winning the World Cup are 7 to 5, what is the probability that they will win the World Cup?

56. If the odds against the U.S. men’s volleyball team defeating China in the Summer 2008 Olympic Games is 9 to 3, what is the probability that the United States will defeat China?

57. Suppose the probability that the Yankees will win the World Series is 0.30.

   a. What are the odds in favor of the Yankees winning the World Series?

   b. What are the odds against the Yankees winning the World Series?

58. Suppose the probability that Rags to Riches will win the Triple Crown in horse racing is 0.15.

   a. What are the odds in favor of Rags to Riches winning the Triple Crown?

   b. What are the odds against Rags to Riches winning the Triple Crown?

59. In the New York Lotto, the player must correctly pick six numbers from the numbers 1 to 59. What are the odds against winning this lottery?

60. Go to the National Safety Council’s Web site at www.nsc.org to find the odds against you being killed by lightning in your lifetime. How many more times likely is it that you will be killed by lightning than win the New York Lotto?

Communicating Mathematics

61. Suppose that we want to assign the probability of $E$ empirically. Complete the following equation: $P(E) =$

62. What condition must we have in order to use the formula $P(E) = \frac{n(E)}{n(S)}$?

63. In the pea plant example, we found that all the first-generation plants had yellow seeds. Why was this the case?

64. If the odds against an event are $a$ to $b$, what are the odds in favor of the event?

65. Explain the difference between an outcome and an event.

66. When rolling two dice, describe in words an event that is the empty set.

67. When rolling two dice, describe in words an event that is the whole sample space.

68. Can an event have just one single outcome in it? Give an example.

69. Explain the difference between the probability of an event and the odds in favor of the event.

70. You know that the probability of an event can never be greater than 1. Can the odds in favor of an event ever exceed 1? Explain.

Using Technology to Investigate Mathematics

71. See your instructor for a tutorial on using graphing calculators to demonstrate some of the ideas we have discussed in this section. Report on your findings.*
72. Search the Internet to find applets that illustrate some of the ideas we have discussed in this section. Report on your findings.

For Extra Credit
In Exercises 73 and 74, assume that to log in to a computer network you must enter a password. Assume that a hacker who is trying to break into the system randomly types one password every 10 seconds. If the hacker does not enter a valid password within 3 minutes, the system will not allow any further attempts to log in. What is the probability that the hacker will be successful in discovering a valid password for each type of password?

73. The password consists of two letters followed by three digits. Case does not matter for the letters. Thus, Ca154 and CA154 would be considered the same password.

74. The password consists of any sequence of two letters and three digits. Case does not matter for the letters. Thus, B12q5 and b12Q5 would be considered the same password.

75. Find some examples of advertising claims in the media. In what way are these claims probabilities?

76. a. Flip a coin 100 times. How do your empirical results compare with the theoretical probabilities for obtaining heads and tails?

b. Roll a pair of dice 100 times. How do your empirical results compare with the theoretical probabilities for rolling a total of two, three, four, and so on?

c. Toss an irregular object such as a thumbtack 1,000 times. After doing this, what probability would you assign to the thumbtack landing point up? What about point down? Does it matter what kind of thumbtack you use? Explain.

77. Investigate other genetic diseases, such as Tay-Sachs or Huntington’s disease. Explain how the mathematics of the genetics of these diseases is similar or dissimilar to the examples we studied in this section.

78. Experiment with an object such as a mouse pad. Slide it off a table 100 times and record how often it lands face down. If the object slips off the table, what is the probability that the object will land face down?

79. Simulate the lost socks example discussed in the Math in Your Life box by doing the following. Take 20 three-by-five cards. Label two of them “pair one,” two of them “pair two,” two of them “pair three,” and so on. Put the 20 cards into a box and draw cards randomly, without replacement, until you have drawn two cards that have the same label. Do this 30 times. On average, how many cards do you draw before you have drawn two cards with the same label?