Introduction

(Last updated on January 15, 2010)

**Digits**: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are all digits.

**Positive integers**: 0, 1, 2, 3, 4, 5, ……20, 21, …are positive integers. These numbers are also called the **whole** numbers.

**Negative integers**: ……-3, -2, -1 are negative integers.

**Natural numbers**: 1, 2, 3, 4, 5, ……20, 21, …are natural numbers.

**Prime numbers**: 2, 3, 5, 7, 11, 13, 17, 19, … are prime numbers, each having two distinct factors, the number itself and 1.

**Rational numbers**: Numbers of the form \( \frac{p}{q} \), \( q \neq 0 \) are called rational numbers. 1, 2, -5, 2/3, 4/7, … are rational numbers.

**Irrational numbers**: \( \sqrt{2}=1.414213562……., \pi=3.14159265……. \) are irrational numbers. Irrational numbers cannot be expressed in the form \( \frac{p}{q} \). When written as decimals, irrational numbers neither terminate nor have a repeating pattern.

Real numbers: The set of real numbers consists of both rational and irrational numbers. Hence it is represented by entire number line.

**Time line**: Let us assume one week is the age of the Earth. Consider the Earth’s 5 billion year age to one week’s worth of time. Suppose also that 700 million years ago, the first fossil animals appeared. The question is when did this “700 million years ago” occur in the course of our week? Our time line is then

\[
\frac{700,000,000}{5,000,000,000} \text{ week} = \frac{7}{50} \text{ week} = \frac{7 \text{ days}}{1 \text{ week}} = \frac{49}{50} \text{ days} \approx 1 \text{ day ago}
\]

**Scientific Notation**: A positive number is written as the product of a number greater than or equal to one but less than ten multiplied with an integer power of ten. The scientific notation of the number 23400000 is \( 2.34 \times 10^7 \) while the scientific notation for the number 0.000000234 is \( 2.34 \times 10^{-7} \).

**Set**: A **set** is just a collection of well-defined and well-distinguished things. We can talk about the set of even numbers, the set of people who are taking algebra or the set of books about large white whales. The things in the set are called the **elements** of the set.

For example, let A be the set of people who served as President of the United States during the 1990s and B the set of all Presidents of the U.S. Then Bill Clinton would be an
element of both set A and set B while George Washington would only be an element of set B.

If we have two sets C and D such that everything in C is in D, we say C is a **subset** of D. Thus in the previous paragraph, A would be a subset of B.

To actually list a set, we use braces, { and } and separate the elements with commas. Thus the set A above could be written as {George Bush, Bill Clinton}.

One important set is the **empty set** (or **null set**), which is the set with nothing in it. For example, if we let C be the set of all people who became president before the age of 18, C would be the empty set.

**Venn Diagrams and Set Operations**

A Venn Diagram is often used to show relationships between sets. At right is a Venn Diagram for two sets A and B. Anything inside the circle labeled A is considered part of set A and similarly for B.

![Venn Diagram](image)

We shade in the section of the diagram we are interested in, so in the figure at left, we have shaded the area that is in both A and B. The **intersection** of two sets is the set with those elements that are in both sets. We denote intersection with an upside down U.

In the diagram at right, we have taken everything in A and added everything that is also in B: the **union** of A and B. The union of two sets is set with those elements that are in the first set, or in the second set, or both sets. For this reason, people associate the word **or** with unions while they associate the word **and** with intersections. We denote union with a simplified U.

Another useful operation is to take the **complement** of a set. The complement of a set is a set with elements those are not in A. So if A is the set of people who are male and B is the set of people who are from Boston. The complement of A would be the set of people who are female. Note that when thinking about complements, we don't pay any attention to any other sets we might have defined. Even though I have a set B defined, when thinking about the complement of A, I only look at A. We denote the complement of a set by a superscript c and associate the word "not" to complement. So "not B" would be the same as the complement of B.
**Your turn:** Suppose we were only interested in the whole numbers from 1 up to 8 and we let $A = \{2, 3, 5, 7\}$, $B = \{2, 4, 6, 8\}$, $C = \{6\}$ and $D = \{3, 4, 5, 6\}$. The universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Determine $A \cap B \cup \overline{C}$. Make the Venn diagram for $A \cap B \cup \overline{C}$.

**Applications with Venn Diagrams**

One place where Venn diagrams are useful is for quickly organizing data into two, possibly overlapping groups. For instance: suppose I knew that in a room were 50 people, 32 of whom were women and 38 of whom were doctors. Of the women, 75% were doctors. How many weren't doctors?

To show this in a Venn diagram, let’s make $A$ represent the people who are women, $B$ those who are doctors. We start by noting that the 50 refers to the whole diagram, the 32 represents all of the circle of $A$, while the 75% shows which fraction was also in $B$. Thus 24 (=75% of 32) of the people were in both $A$ and $B$. Giving us the information at right.

Since all of $A$ is 32 and we have 24 so far, we know we must have 8 left over for that part $A$ that isn't $B$ ($A$ intersected with the complement of $B$).

Now we use the fact that the total number of $B$ was 38. We have 24 so far in $B$, so we need 14 more.

Finally, we know there were 50 people in the room. We have 46 so far, that leaves 4 who must be outside both $A$ and $B$, hence people who were not female (so male) and not doctors. Thus four men weren't doctors.

Suppose instead I asked how many people were men or doctors? This means we are taking the union of the complement of $A$ and $B$, hence $4 + 14 + 24 = 42$. 