Chapter 8 Geometry

We will discuss following concepts in this chapter.

Two Dimensional Geometry: Straight lines (parallel and perpendicular), Rays, Angles (acute, obtuse, straight, vertical, complement, supplementary), Similar and congruent shapes, Polygons (Triangles (scalene, equilateral, isosceles, right, obtuse angled, acute angled), Quadrilateral (parallelogram, rectangles, squares, rhombus, trapezoids), Pentagons (5 sided polygons), Hexagons (6 sided polygons), Heptagons (7 sided polygons), Octagons (8 sided polygons), Nonagons (9 sided polygons), Decagons (10 sided polygons)), Regular polygons, Circle (radius, diameter, circumference), Area, Perimeter.

Geometry of Three Dimensions: Volume and surface area of box, cube, Polyhedron (prism, pyramid, cone), cylinder, sphere.

Trigonometry: soh, cah, toa from a right triangle.

Triangle: Let $a$, $b$, $c$ be the measures of three sides and $h$ be the height from a vertex to the side $b$ of a triangle. Then

\[
\text{Perimeter} = a + b + c \text{ units, and semi-perimeter } s = \frac{(a + b + c)}{2} \text{ units}
\]

\[
\text{Area} = \frac{1}{2} \cdot b \cdot h \text{ square units. Area (in terms of sides)} = \sqrt{s(s-a)(s-b)(s-c)} \text{ square units}, this formula is known as Herron’s formula.
\]

Quadrilateral: Let $a$, $b$, $c$, $d$ be the sides of a quadrilateral, then perimeter $= a + b + c + d$

\[
\text{Area of parallelogram} = \text{base} \times \text{height}
\]

\[
\text{Area of rectangle} = \text{length} \times \text{width}
\]

Circle: Circumference $= 2\pi r$ and Area $= \pi r^2$, we have also $\pi = \frac{\text{circumference}}{\text{diameter}}$

Oval (Ellipse): Area $= \pi ab$, where $a$ and $b$ are the lengths of the semi-major and semi-minor axes.

Volume and surface area:

Box (rectangular solid): Let $a$, $b$, $c$ be the length, width, and height of a box. The surface area of the box is equal to $S = 2(ab+bc+ca)$ ands the volume is $V = a \times b \times c$

Volume of a prism or cylinder $V = A \times h$, where $A$ is the area of the base and $h$ is the height.

Surface area of a closed cylinder $S = 2\pi r + 2\pi r^2$

Volume of a pyramid or a cone $V = \frac{1}{3} A \times h$, where $A$ is the area of the base and $h$ is the height.

Volume of a sphere $V = \frac{4}{3} \pi r^3$, where $A$ is the area of the base and $h$ is the height.

Surface area of sphere $S = 4\pi r^2$
**Trigonometry**: soh, cah toa stands for the following results

The right triangle has hypotenuse $h$, opposite $o$ and adjacent $a$

Then $\sin x = \frac{o}{h}$ and $\cos x = \frac{a}{h}$ and $\tan x = \frac{o}{a}$

Important relations in right triangles: Angles are in degrees.

**Different Positive Angles**: Acute angle, Right angle, Straight angle, Obtuse angle and Reflex angle

<table>
<thead>
<tr>
<th>Types of Angle</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute Angle</td>
<td>An angle, which is less than 90 degree or less than $\frac{\pi}{2}$</td>
<td>60°, 10.55°, $\frac{\pi}{6}$, $\frac{\pi}{3}$</td>
</tr>
<tr>
<td>Right angle</td>
<td>An angle equal to 90 degree or $\frac{\pi}{2}$ is right angle</td>
<td>90 degree or $\frac{\pi}{2}$</td>
</tr>
<tr>
<td>Straight angle</td>
<td>An angle equal to 180 degree or $\pi$</td>
<td>180 degree or $\pi$</td>
</tr>
<tr>
<td>Obtuse angle</td>
<td>An angle greater than 90 degree but less than 180 degree</td>
<td>100°, 93.55°, $\frac{2\pi}{3}$</td>
</tr>
<tr>
<td>Reflex angle</td>
<td>An angle, which is greater than 180 degree but less than 360 degree</td>
<td>200°, 193.55°, $\frac{3\pi}{2}$</td>
</tr>
</tbody>
</table>

**Trigonometric angles**

Two rays drawn from a common vertex form an angle. One ray of the angle is called the initial side while the other is referred to as the terminal side. The angle formed is identified by showing the direction and amount of rotation between initial and terminal sides. If the rotation of terminal side from initial side is in the counter clockwise direction we call it positive angle. If the rotation is in clockwise direction the angle is negative.
Angle measured in degrees

The angle from between initial and terminal sides by one complete revolution is equal to 360 degrees, we write as $360^\circ$, which is equal to 4 right angles. Thus one right angle is equal to $90^\circ$. On the other hand one straight angle is equal to 2 right angles or $1/2$ of a complete revolution. Thus one straight angle is equal to $180^\circ$.

Angle measured in radians

In the radian system of measuring an angle, the measure of one revolution is $2\pi = 360^\circ$. Thus $\pi = 180^\circ = \text{right angle}$, and $\pi/2 = 90^\circ = \text{right angle}$.

A central angle is an angle whose vertex is at the center of a circle. If the central angle formed between the rays is equal to the radius of the circle, we call it one radian.

Relation between degree and radian measures
We have found that \(2\pi\) radian = 360 degrees. Now we have 1 radian = \(\frac{360}{2\pi}\) = \(\frac{180}{\pi}\)
where \(\pi = 3.14159265359\). Also 1 degree = \(\frac{2\pi}{360}\) = \(\frac{\pi}{180}\).

**Conversion between degrees and radians**

- To convert degrees to radians, multiply given degrees by \(\frac{\pi}{180}\) and your result is in radians
- To convert radians to degrees, multiply given radians by \(\frac{180}{\pi}\) and your result is in degrees

**The meaning and value of \(\pi\)**

Suppose the measure of circumference is \(C\) and the diameter is \(D\) then

\[
\frac{C}{D} = \pi \approx 3.142
\]

The meaning of \(\pi \approx 3.142\) is that the circumference of the circle is little more than 3 times the diameter.

**Examples:**

1. Convert 720° to radian measure.
2. Find the positive coterminal angles of 730°
3. Convert \(6\pi\) to degree measures

**Solutions:**

1. Multiply by \(\frac{\pi}{180}\) to get \(720 \times \frac{\pi}{180} = 4\pi\) radians
2. The positive coterminal angles of 730° are 10°, and 370°
3. Multiply by \(\frac{180}{\pi}\) to get \(6\pi \times \frac{180}{\pi} = 1080\) degrees
Area of a sector of a circle

Consider a circle of radius $r$. If $\theta$ is measured in **radian** is the central angle of the circle, then the area of the section $OAB$ is given by

$$\text{Area of section } OAB = \frac{1}{2} r^2 \theta \text{ square unit.}$$

1. The radius of a circle is given 10 cm. Find the area of the section formed by the initial and terminal sides with central angle 30 degrees.

**Solution:** The area of the section is $OAB = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 10^2 \times \frac{30\pi}{180} = 26.18 \text{ cm}^2$

Some standard values of sine, cosine and tangent functions: Let us produce a table, which is useful in this case.

How do we make the first row:

- Write successively 0 1 2 3 4
- Divide each number by 4
- Take square root
- The values are for sine function
- Write sine values from backward for cosine function
- Divide sine values by corresponding cosine values to get values for tangent function.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>$\frac{1}{2}$</th>
<th>$\frac{\sqrt{2}}{2}$</th>
<th>$\frac{\sqrt{3}}{2}$</th>
<th>1</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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<td>$\frac{\pi}{4}$</td>
<td>$\frac{\pi}{3}$</td>
<td>$\frac{\pi}{2}$</td>
<td>1</td>
</tr>
<tr>
<td>sin</td>
<td>0</td>
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<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>1</td>
</tr>
<tr>
<td>cos</td>
<td>1</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>tan</td>
<td>0</td>
<td>$\frac{1}{\sqrt{3}}$</td>
<td>1</td>
<td>$\sqrt{3}$</td>
<td>Undefined</td>
</tr>
</tbody>
</table>
Exercise Set

1. Find the supplementary angle of $35^\circ$.

2. For a square whose diagonal is $\sqrt{2}$, find perimeter and the area.

3. A ladder that is 9.5 feet long casts a shadow that is 19 feet long at the same instant that a tree casts a shadow that is 100 feet long. How tall is the tree?

   (Solution: use similar triangle notion. $\frac{9.5}{19} = \frac{x}{100}$ then $x = 50$, the tree is 50 feet tall.)

4. A smaller right circular cylinder is cut out of a larger solid that is also a right circular cylinder, but 2 inches about its base, as shown below. Find the volume of the remaining solid.

5. The oval is inside of a square that is not drawn to scale. Find the area of the shaded region.

6. Draw the figure of a propane gas tank that consists of a cylinder with a hemisphere at each end. The overall length of the tank is 10 feet and the diameter of the cylinder is 4 feet. Find the volume of the tank.

7. A water storage tank is an upside down cone. If the diameter of the circular top is 12 feet and the length of the sloping side is 10 feet, how much water will the tank hold? Graph the tank showing length of the diameter and the sloping side.

8. Find the area of the triangle. The angles are in degrees.
9. The center of the bigger circle is O, the medium circle is P and the smaller circle is Q, all lie on the line AB. The small circle passes through P and the medium circle passes through O. Place O, P and Q in the appropriate place and find the ratio of the area of the entire shaded region to the area of the white region.

10. In the figure at the right, C is the center of the circle. What is the value of c?

11. In the figure below, C is the only point that the triangle $ABC$ and square $CDEF$ have in common. What is the value of $a+b$?

12. In the figure below, what is the value of $b$?

13. In the figure below, if $x$ is 150 more than $y$, what is the value of $y$?
14. In the figure below, a small square is drawn inside a large square. If the shaded area and the white area are equal, what is the ratio of the side of the large square to the small square?

15. Of the figure below, container I is a rectangular solid whose base is a square 4 inches on a side, and container II is a cylinder whose base is circle of diameter 4 inches. The height of each container is 5 inches. How much more water, in cubic inches, will container I hold than container II?

16. In parallelogram ABCD above, what is the value of $x$?

17. In parallelogram ABCD above, what is the value of $x$?

18. Three lines are drawn in a plane; of three lines two are parallel. Which of the following CANNOT be the total number of points of intersection?

A. 0  (B) 1  (C) 2  (D) They all could  (E) None given
19. The degree measure of each of three angles of a triangle is an integer. Which of the following CANNOT be the ratio of their measure?
   (A) 2:3:4    (B) 3:4:5    (C) 4:5:6    (D) 5:6:7    (E) 6:7:8

20. Let A, B, and C be three points in a plane such that AB:BC = 3:5, which of the following can be the ratio AB:AC?
   i. 1:2  
   ii. 1:3  
   iii. 3:8  
   (A) I only    (B) II only    (C) III only    (D) I and III only    (E) I, II, and III

21.  
   ![Diagram of a rectangle ABCD with a shaded quadrilateral BDEF]

   If a point is chosen at random from the interior of rectangle ABCD above, what is the probability the point will be in the shaded quadrilateral BDEF?
   (A) 1/4    (B) 1/3    (C) 5/12    (D) 1/2    (E) 7/12

22.  
   ![Diagram of a triangle ABC with a line segment BD]

   Which of the following could be a value of $x$, in the diagram above?
   (A) 10    (B) 20    (C) 40    (D) 50    (E) Any of the given

23. In the picture, the three circles are tangent to one another. If the ratio of the diameter of the large circle to the diameter of the small white circle is 3:1, what fraction of the largest circle has been shaded?
24. Find the area of the shaded region:

The figure above consists of semi circle, regular hexagon, an oval and three triangles. The scale on axis and y axis both are in one cm a unit.