Simple Interest:
Total simple interest on principal $P$ is $I = Prt$ and Amount $A = P + Prt = P(1 + rt)$

Compound Interest:
Amount $A = P\left(1 + \frac{i}{n}\right)^n$ and total compound interest $CI = A - P$

Annual yield $AY = \left(1 + \frac{i}{n}\right)^n - 1$ which is always greater than or equal to the CI for the same situation.

The magic number 72: Time to double an amount at $r\%$ compounded yearly $= \frac{72}{r}$ years

Future Value or Amount of Annuity $A = pymt \left[\frac{(1 + i/n)^n - 1}{i/n}\right]$ and

Present value of annuity $V = wd \left[\frac{1 - (1 + i/n)^{-nt}}{i/n}\right]$

Withdrawal $wd = V \left[\frac{i/n}{1 - (1 + i/n)^{-nt}}\right]$

We have solved almost all the problems in class. Review the problems.

Example 1a. Suppose you invest $42000 at 5.5\%$ simple interest for $\frac{5}{3}$ years. Find how much interest is found in the account. Find also how much is found in the account. (Answer: 12319.92, 54319.92)

Example 1b. Suppose you invest $42000 at 5.5\%$ simple interest for 320 days. Find how much interest is found in the account. Find also how much is found in the account.

Example 2. Suppose you invest $42000 at $\frac{5}{3}$\% simple interest for $\frac{5}{4}$ years. Find how much interest is found in the account. Find also how much is found in the account.

Example 3. Suppose you invest some amount at $\frac{5}{3}$\% simple interest for $\frac{5}{4}$ years. The total interest found is $550. Find how much you have invested.

Example 4. Suppose you invest some amount at $\frac{7}{3}$\% simple interest for $\frac{7}{8}$ years. The total interest found is $5050. Find how much you have invested the present value).
Example 5. Find the amount of money that must be invested now at $5\frac{3}{4}\%$ simple interest so that it will be worth $1000$ in 2 years. (Answer $896.86$)

Example 6. Add-on-interest: ABC company purchase $1300$ worth of kitchen appliances at CD Mart. They put $200$ down and agree to pay the balance at a $10\%$ add-on-interest loan for 2 years. Find the monthly payment.

Solution: Add-on-interest is a simple interest loan. We use $A = P(1 + rt)$. $P = 1300 - 200$ Find that $A = 1320$ and the monthly payment for 2 years is $1320/24 = 55$

Example 7. Credit card finance charge: The activity of a credit card account for one billing cycle is shown below. Find the average daily balance and the finance charge if the billing period is October 15 through November 14. The previous balance was $346.57$ and the annual interest rate is $21\%$.

<table>
<thead>
<tr>
<th>Time interval</th>
<th>Days</th>
<th>Daily balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct 15 – 20</td>
<td>6</td>
<td>$346.57$</td>
</tr>
<tr>
<td>Oct 21 – 22</td>
<td>2</td>
<td>$346.57 - 50 = 296.57$</td>
</tr>
<tr>
<td>Oct 23 – Nov 6</td>
<td>15</td>
<td>$296.57 + 42.14 = 338.71$</td>
</tr>
<tr>
<td>Nov 7 – 14</td>
<td>8</td>
<td>$338.11 + 18.55 = 357.26$</td>
</tr>
</tbody>
</table>

Average daily balance = $\frac{6 \times 346.57 + 2 \times 296.57 + 15 \times 338.71 + 8 \times 357.26}{31} = 342.30$ dollars

Total interest = $I = Prt = 342.30 \times 0.21 \times \frac{31}{365} = 6.11$

Example 5. Suppose you invest $42000$ at $5\%$ compound interest for 5 years. Find how much interest is found in the account after 5 years. Find also how much you have in the account after 5 years. Find annual yield and explain what is meant by the annual yield.

Example 6. Suppose you invest $42000$ at $5\%$ interest compounded biweekly for 5 years. Find how much interest is found in the account after 5 years. Find also how much you have in the account after 5 years. Find annual yield and explain what is meant by the annual yield.

Example 7. Suppose you invest $42000$ at $7\frac{6}{7}\%$ interest compounded quarterly for 5 years. Find how much interest is found in the account after 5 years. Find also how much you have in the account after 5 years. Find annual yield and explain what is meant by the annual yield.
Example 8. How long does it take for a sum of money to be doubled at $7\frac{6}{7}$% interest compounded yearly?

Example 9. How long does it take for a sum of money to be doubled at $5\frac{1}{4}$% interest compounded quarterly?

Example 10. When Joe was born, his father deposited $3000 into a special account for Joe’s college education. The account earned 6.5% interest compounded daily. Find how much will be in the account when Joe is 18 years. If on turning 18, Joe arranges for monthly interest to be sent to him, how much will he receive each 30-day month?

Answer: 9664.97, 51.77

Annuities

Example 1. Find the amount of an IRA annuity after 6 years if you paid into it $50 per month at 12% compounded monthly. (Answer: $5235.50)

Example 2. Find the amount of an IRA annuity after 6 years if you paid into it $50 every quarter at 12% compounded quarterly. (Answer: $1721.32)

Example 3. Find the amount of an IRA annuity after 6 years if you paid into it $50 per month at 12% compounded quarterly. (Answer: $5163.97)

Example 4. Find the amount of an IRA annuity after 6 years if you paid into it $50 per week at 12% compounded monthly. (Answer: $20941.99)

Example 5. If you pay $50 each month into an extended Christmas club account, paying 8.5% interest compounded monthly, what amount do you have after 8 months? How much did you put into the account? (Answer: $410.01, $400)

Example 6. The amount required is $50000 after 10 years at 7% compounded semiannually. What is the semiannual payment? (Answer: $1768.05)

Example 7. Find the present value of an annuity if the withdrawal is $2000 per month for 3 years at 4% compounded monthly. (Answer: $67741.53)

Example 8. Find the present value of an annuity if the withdrawal is $2500 per month for 3 years 4 months at 5% compounded quarterly. (Answer: $91585.17)

Example 9. You make $100 payments each month into an annuity for 20 years at $7\frac{3}{4}$% annual compounded monthly. After 20 years, you deposit the entire amount in the account in a present day annuity that earns $8\frac{1}{2}$% compounded monthly. How much
could you withdraw each month if you needed the money to live off of for another 20 years? (Answer: $57105.30, $495.57)

**Example 10.** You make $150 payments each month into an annuity for 15 years at $\frac{7}{4}\%$ annual compounded monthly. After 15 years, you deposit the entire amount in the account in a present day annuity that earns $\frac{8}{2}\%$ compounded monthly. How much could you withdraw each month if you needed the money to live off of for another 15 years? (Answer: $73419.05, 722.99$

**The Mortgage Payments**

For mortgage payments we use the formula 
\[
pymt = \frac{(\text{loan amount})i/n}{1-(1+i/n)^{-m}}
\]

Monthly Interest = $\frac{p_t}{12}$, $p_t$ is the outstanding balance for the month $t$.

**Example 11.** In November of 2003 the maximum amount of money one could borrow under the terms of conventional loan was $322,700.00. Find the monthly payment for the maximum conventional loan if you received the loan and agreed to repay the money over the next 30 years at an APR of $\frac{5\frac{3}{8}}{8}\%$ convertible monthly.

**Solution:**
\[
pymt = (322700) \frac{0.05375/12}{1-(1+0.05375/12)^{-12*30}} = 1807.03
\]

**Example 12.** In November of 2003 the maximum amount of money one could borrow under the terms of conventional loan was $322,700.00. You received the loan and agreed to repay the money over the next 30 years at an APR of $\frac{5\frac{3}{8}}{8}\%$ convertible monthly.

a) What dollar amount for the first payment goes to the interest?
b) Find the principal portion from the first payment.
c) Find the outstanding balance after first monthly payment for the maximum conventional loan
d) What dollar amount goes to the interest from the second payment? Find the principal portion also.

**Solution:** From Example 11, pymt = (322700) \[
\frac{0.05375/12}{1-(1+0.05375/12)^{-12*30}} = 1807.03
\]
a) The interest portion on the first payment = $pi = 322700(0.05375/12) = 1445.43$
b) The principal portion from the first payment = $1807.03 - 1445.43 = 361.60$
c) The outstanding balance after 1 payment = $322700 - 361.60 = 322338.40$
d) The interest due on the second payment = $322338.40(0.05375/12) = $1443.81 and principal portion = 1807.03 – 1443.81 = $363.22

e) A $400,000 mortgage that has 20% down payment and rest has been financed over 30 years at 6.5% APR on amortized loan.

   a) Find the monthly payment.

   b) Find the outstanding balance after the first payment.

   c) Find the interest portion from the first payment of the loan.

   d) Find the principal portion of the payment, which reduces the outstanding balance.

   e) Find outstanding balance after the second payment

   f) Find the interest portion from the second payment of the loan

   g) Find the principal portion from the second payment of the loan.