Chapter 1. The Logic

Section 1.1 Deductive versus Inductive Reasoning

What is logic: The science of correct reasoning. Logic is fundamental both to critical thinking and problem solving.

What is reasoning: Reasoning is defined as the drawing of inferences or conclusions from known or assumed facts.

Deductive versus Inductive reasoning: The type of logic, which is known as deductive reasoning is the application of a general statement to a specific instance.

Example of deductive reasoning: To solve any quadratic equation like $x^2 - 2x + 1 = 0$, one can use the general formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for the equation $ax^2 + bx + c = 0$.

On the other hand, in inductive reasoning conclusions may be probable but not guaranteed.

Example of inductive reasoning: The probable answer of next term of the sequence 1, 8, 15, 22, 29 could be 36. But it is not guaranteed. One may answer that the next number in the sequence is 5, if he/she thinks the numbers are coming from Mondays in August 2005. Then next Monday is on September 5. We can only use inductive reasoning and give one or more possible answers.

What is argument: The standard dictionary meaning of argument is a discussion in which there is a disagreement.

Valid versus invalid argument: If the conclusion of an argument is guaranteed, the argument is valid. On the other hand if the conclusion of the argument is not guaranteed, the argument is invalid.

Saying that an argument is valid does not mean that the conclusion is true: We verify the situation by an example. Consider two premises 1. All doctors are men, 2. My mother is a doctor. Then the valid argument “My mother is a man” is not a true conclusion.

Saying that an argument is invalid does not mean that the conclusion is false. We verify the situation also by an example. Consider two premises 1. All professional wrestlers are actors, 2. The Rock is an actor. Then the invalid argument “the Rock is a professional wrestler”, may not be false. We will verify valid and invalid arguments and conclusions with Venn diagram.

What is a Venn Diagram: A Venn diagram consists of a rectangle, representing the universal set, and various closed figures within the rectangle, each representing a set.
Venn diagram and invalid arguments: To show that an argument is invalid you must construct a Venn diagram in which the premises are met yet the conclusion does not necessarily follow.

Example 1. Construct a Venn diagram to determine the validity of the given argument.

- No snake is warm-blooded
- All mammals are warm-blooded
- Therefore, snakes are not mammals

Solution: Suppose \( x \) represents snakes. The position of \( x \) in the diagram is unique and shows that the argument is valid.

Example 2. Construct a Venn diagram to determine the validity of the given argument.

- All professional wrestlers are actors
- The Rock is an actor
- Therefore, the Rock is a professional wrestler

Solution: Suppose \( x \) represents Rock. Then the different position of \( x \) in the diagram shows that the argument is invalid.

Exercise 3. a) Construct a Venn diagram and verify that the following argument is invalid.

1. (Major premise) Some plants are poisonous
2. (Minor premise) Broccoli is a plant
Therefore (conclusion) Broccoli is poisonous.

b) Verify that the argument in question 1 is deductive, but argument in question 2 is inductive.

Section 1.2 Symbolic logic

What is a statement (or proposition): A statement is a sentence that is either true or false. All logical reasoning is based on statement.
Examples of statements (or propositions).
1. Apple manufactures computers.
2. A $2000 computer that is discounted 25% will cost $1500 (true)
3. A $2000 computer that is 25% discounted will cost $1000 (false)

Examples which are not statements.
1. I am telling the truth (either true or false)
2. Apple manufactures the world’s best computers (either true or false)
3. Did you go to market yesterday? (question)

Symbols for logic: By tradition symbolic logic uses lowercase letters as labels for statements. The most frequently used letters are $p, q, r, s,$ and $t$.

Compound Statements and Logical Connectives
A compound statement is a statement that contains one or more simpler statements. A compound statement can be formed by inserting the word ‘not’ into a simpler statement or by joining two or more simpler statements with connective words such as ‘and’, ‘or’, ‘if …then…..’, ‘only if’, ‘if and only if’ etc. The compound statement could be a negation, a conjunction, a disjunction, a conditional, or any combination thereof.

Negations: The negation of a statement is the denial of the statement and is represented by the symbol $\sim$. For example, given the statement $p$: it is snowing, the negation is $\sim p$: it is not snowing.

Negations of statements containing qualifiers. The words some, all, no (or none) are referred to as qualifiers. The negation of “all $p$ are $q$ would be some $p$ are not $q$” and the negation of “some $p$ are $q$ would be no $p$ are $q$”.
One may remember the following diagram.

![Negation Diagram]

Example 4. Determine which pairs of statements are negations of each other.
1. Some of the beverages contain caffeine
2. Some of the beverages do not contain caffeine
3. None of the beverages contain caffeine
4. All of the beverages contain caffeine

Solution: The negation of 1 is 3 and the negation of 3 is 1.
On the other hand the negation of 2 is 4 and the negation of 4 is 2.
**Conjunction:** A conjunction consists of two or more statements connected by the word ‘and’. Suppose \( p \) and \( q \) are two simpler statements, then \( p \land q \) is called the conjunction of \( p \) with \( q \).

**Disjunction:** A disjunction consists of two or more statements connected by the word ‘or’. Suppose \( p \) and \( q \) are two simpler statements, then \( p \lor q \) is called the disjunction of \( p \) with \( q \).

**Conditional:** A conditional consists of two or more statements connected by the words ‘if .. then ..’. Suppose \( p: I \) am healthy, and \( q: I \) exercise regularly are two simpler statements, then \( p \rightarrow q \) is called the conditional of \( p \) with \( q \), which is in word ‘**If I am healthy then I exercise regularly.**

**Only if:** Only if is the conclusion of the conditional. The symbol \( p \rightarrow q \) can be read as “\( p \) only if \( q \)”.

**Biconditional:** A biconditional is a statement of the form \((p \rightarrow q) \land (q \rightarrow p)\) and is symbolized as \( p \iff q \) and read as “\( p \) if and only if \( q \). And sometimes also abbreviated as \( p \iff q \)”.

**Variations of a conditional:** For two given statements \( p \) and \( q \), various conditional statements can be formed. We discuss the following three cases.

**Converse:** The converse of the conditional “if \( p \) then \( q \)” is “if \( q \) then \( p \)”. Symbolically converse of \( p \rightarrow q \) is \( q \rightarrow p \).

**Inverse:** The inverse of the conditional “if \( p \) then \( q \)” is “if not \( p \) then not \( q \)”. Symbolically inverse of \( p \rightarrow q \) is \( \neg p \rightarrow \neg q \).

**Contrapositive:** The contrapositive of the conditional “if \( p \) then \( q \)” is “if not \( q \) then not \( p \)”. Symbolically inverse of \( p \rightarrow q \) is \( \neg q \rightarrow \neg p \).

**Equivalent statements:** A conditional and its contrapositive are equivalent to each other. Symbolically \( p \rightarrow q \equiv \neg q \rightarrow \neg p \).
The converse and inverse of the conditional are equivalent to each other. Symbolically \( q \rightarrow p \equiv \neg p \rightarrow \neg q \).

**Example 5.** Use the symbolic representation
\[
p: I \text{ am innocent} \quad q: I \text{ have an alibi} \quad r: I \text{ go to jail}
\]
to express the symbol \( (p \land \neg q) \rightarrow r \) in words
(The solution is “**If I am innocent and I do not have an alibi then I go to jail**”)

**Section 1.3 Truth Tables**

**Truth Value and Truth Table:** The truth value of a statement (or proposition) is either true (T) or false (F). The truth table of a statement is the classification of the statement as true or false and is denoted by T or F. A convenient way of determining whether a compound statement is true or false is to construct a truth table.
Construction of a truth table.

Note: Remember the three sentences law of logic:

1. The conjunction $p \land q$ is true if both $p$ and $q$ true, otherwise it is false
2. The disjunction $p \lor q$ is false if both $p$ and $q$ false, otherwise it is true
3. The conditional $p \rightarrow q$ is false if $p$ is true and $q$ is false, otherwise it is true

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<th>$p \rightarrow q$</th>
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We consider the following example where truth table is a useful tool to find the solution.

**Example 6.** Under what specific condition(s) is the following compound statement true? “I have a high school diploma, or I have a full time job and no high school diploma.”

We suppose that $p$: I have a high school diploma
$q$: I have a full time job

The symbolic form of the compound sentence “I have a high school diploma, or I have a full time job and no high school diploma” is $p \lor q \land \sim p$

We will make a truth table to find when it is true:

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<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \lor q$</th>
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<th>$p \lor q \land \sim p$</th>
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Conclusion: The statement “I have a high school diploma, or I have a full time job and no high school diploma” is true when statement “I have a high school diploma” is false and the statement “I have a full time job” is true.

**Example 7.** Construct a truth table for the symbol $(p \land q) \rightarrow \sim r$

To construct a truth table one needs to remember

Step 1. The tree diagram for the symbols (simple statement) $p$, $q$, and $r$

```
  T
 / \
/   \
F   F

  T
 / \
/   \
F F

  T
 / \
/   \
F F

  F
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T T
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Step 2. Last column in tree diagram contains the entries for column $p$, $q$, and $r$ of the truth table. We need 8 rows and 6 columns in the truth table.
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Step 3. Find negation for \( r \)
Step 4. Remember that \( p \land q \) is true (T) if both \( p \) and \( q \) true (T)
Step 5. Remember that \((p \land q) \rightarrow \neg r\) is false (F) if \((p \land q)\) is true (T) but \( \neg r \) is false (F).

**Question 1.** Construct a dictionary and a truth table to determine if the following statements are logically equivalent
i) It is Friday and I receive a paycheck
ii) It is not Friday and I do not receive a paycheck

**Question 2.** Write in sentence form what is the converse, inverse and contrapositive of the statement “If I receive a paycheck, then it is Friday.”

**Question 3.** What is a Tautology? If a statement is a tautology, what can you say about the statement?

**Question 4.** Define the necessary symbols and rewrite the following arguments in symbolic form:
1. If the defendant is innocent, the defendant does not go to jail
2. The defendant does not go to jail
Therefore the defendant is innocent

**Question 5.** Use the following dictionary of symbols
\( p \): The defendant is innocent
\( q \): The defendant does not go to jail
Is the symbol \((p \rightarrow q) \land q\) \( \rightarrow p\) a tautology? Write your conclusion in word.

**Question 6.** Prove the De Morgan’s Laws
a) For two simple statements \( p \) and \( q \) \( \neg (p \land q) \equiv p \lor \neg q \)
b) For two simple statements \( p \) and \( q \) \( \neg (p \lor q) \equiv p \land \neg q \)

**Question 7.** Use De Morgan’s law to find the negation of each of the following:
a) It is Friday and I receive a paycheck
b) You are correct or I am crazy
c) The streets are wet or it is not raining
d) I have a college degree and I am not employed
e) It is snowing and classes are cancelled
f) Maria is a doctor a Republican

Dr. Firoz
Practice exercise problems
Make Venn diagram and check for validity:

1. (1) All homeless people are unemployed. Answer: Invalid
   (2) Bill Gates is not a homeless person
   Therefore, Bill Gates is not unemployed

2. (1) All homeless people are unemployed. Answer: Valid
   (2) Bill Gates is not unemployed
   Therefore, Bill Gates is not a homeless person

3. (1) All poets are loners. Answer: Valid
   (2) All loners are taxi drivers
   Therefore, all poets are taxi drivers

4. (1) All roads lead to Rome Answer: Valid
   (2) Route 66 is a road
   Therefore, Route 66 leads to Rome

5. (1) All forest rangers are environmentalists Answer: Invalid
   (2) All forest rangers are storytellers
   Therefore, all environmentalists are storytellers

6. Determine if the argument below is valid or invalid, deductive or inductive
   (1) The television set did not work three nights ago. Answer: Inductive
   (2) The television set did not work last night
   Therefore, the set is broken

7. Fill in the blank with what is most likely to be the next two numbers, explain your reasoning:

   12, 5, 10, 3, -----, ----
   0, 12, 24, 36, -----, ----
   0, 12, 60, 252, -----, ----- 
   3, 3, 6, 9, 15, -----, --------
   7, 10, 13, 8, 11, -----, ----- 
   5, 8, 11, 2, -----, ----, ----

8. Check which are the statements
   (1) Is $\sqrt{2}$ a rational number?
   (2) Abraham Lincoln was the best president
   (3) His name is David
   (4) Apple is the best computer
   (5) Did you complete your home work?
   (6) Two plus two is four
   (7) Two plus two is 8
9. Chose the sentence that represents the negation of each statement

Statement 1: Her dress is not red
   - None of her dress is red
   - Some of her dress is not red
   - Her dress is red
   - All her dress is not red

Statement 2: No sleeping bag is waterproof.
   - All sleeping bags are waterproof
   - Some sleeping bags are not waterproof
   - Some sleeping bags are waterproof
   - No sleeping bags are not waterproof

10. Using symbolic representation choose the best way to express each compound statement in symbolic form:

   \[ p: \text{A person plays the guitar} \quad q: \text{A person rides the motorcycle} \]
   \[ r: \text{A person wears the leather jacket} \]

   (a) If the person plays the guitar or rides a motorcycle, then the person wears a leather jacket.

   (b) A person wears the leather jacket and does not play the guitar or ride a motorcycle.

11. Make a truth table for \( p \land q \rightarrow r \lor s \) and \( p \land q \rightarrow (r \lor s) \)

12. Explain what is meant by tautology. Show that \( (p \land q) \rightarrow (p \lor q) \) is a tautology, where \( p \) and \( q \) are simple statements.

13. Determine whether the following argument is valid:
   1. If the defendant is innocent, the defendant does not go to jail
   2. The defendant does not go to jail

   Therefore, the defendant is innocent.

14. Use truth table show that the conclusion is valid (a tautology):
   1. All lawyers study logic
   2. You study logic only if you are a scholar
   3. You are not a scholar

   Therefore, you are not a scholar

15. Show that \([(p \rightarrow q) \land (r \rightarrow q) \land (s \rightarrow p)] \rightarrow (s \rightarrow r)\) is a tautology.

GOOD LUCK
Group work 1. Name: 1. 2. 3.

1. Make a truth table for \( p \lor q \land \neg p \)

2. Test the validity of the argument given below:

1. If the defendant is innocent, the defendant does not go to jail
2. The defendant does not go to jail

Therefore, the defendant is innocent.

Use: \( p \): The defendant is innocent
\( q \): the defendant goes to jail