We study the product variety and pricing decisions in a supply chain that includes two competing producers who plan to work with middle-tier companies. End customers are characterized by the amount of money they are willing to pay for different quality levels. Products reach the end customers via assembly companies positioned in the middle-tier between the producers and the end customers. We analyze the influence of the assembly companies on the market equilibrium. We consider a multi-leader Stackelberg game between the producers and assembly companies, and compare different supply chain configurations: direct-shipping with no middle-tier, monopoly with a single assembly company, and duopoly with two assembly companies. We find that in the Nash equilibrium producers and assembly companies should choose to differentiate their product offering. The presence of the middle-tier assembly companies can reduce the intensity of competition in the supply chain and increase the revenue for the producer of the low-end product whereas the producer of the high-end product prefers either the direct shipping or a duopoly with product differentiation depending on the quality levels of the products. We show that product differentiation is not always the equilibrium strategy for the assembly companies.

Key words: Product variety, Vertical differentiation, Game theory, Supply chain.

1. Introduction

In today’s competitive environment manufacturers often focus on their core competencies and as a result the manufacturing supply chain comprises multiple stages of production, each stage corresponding to a separate firm. For example, Intel and AMD manufacture semiconductors chips and sell them to Original Equipment Manufacturers (OEMs) such as DELL and HP who then assemble, distribute, and sell computers to the end customers. The OEMs act as a middle tier
between the chip producers and end market. To execute their product and market strategies, upstream producers such as Intel and AMD have to anticipate the influence of the middle-tier players on the market. In this paper, we study how the middle-tier and competition between the middle-tier firms influence the product variety and pricing decisions in a supply chain.

We consider industries in which the end customers are heterogeneous in the sense that they are willing to pay more for products with higher quality. For example, in general, consumers are willing to pay more for a computer with a faster CPU speed, larger memory, and lower energy consumption. These attributes together determine the quality of a chip and lead to an ordering. Notice that quality may stand for a combination of any number of attributes that leads to a linear ordering of products in the assessment by the customers. In the literature the term quality rather than performance has been adopted and we will henceforth refer to quality levels only.

We study two producers that offer a discrete set of vertically differentiated products. We consider a prototypical setting in which there are two products, namely the high-end (H) product and the low-end (L) product. Both producers have the technology and capacity to offer both products. Products typically reach the end customers via assembly companies positioned in the middle-tier between the producers and the end customers. We assume that assembly operations performed in the middle-tier preserve the quality ordering of the vertically differentiated products. The resulting hierarchy is illustrated in Figure 1.c: First, the two producers simultaneously decide which product(s) to offer to the assembly companies and at what prices. Next, assembly companies simultaneously determine which products to offer to the end customers and what prices to charge. Finally, each end customer buys a product that maximizes her utility. We contrast this supply chain with two other supply chain configurations: A baseline given by direct sales by the producer to the end customer (Figures 1.a) and a supply chain with a single assembly company (Figure 1.b).

We address the following issues central to the product variety and pricing problem in a multi-stage supply chain framework:

- What is the best supply chain configuration for each of the three participants: the producers, the assembly companies, and end customers? Does there exist a market equilibrium?
What is the effect of middle-tier companies on product variety and pricing decisions? Should the companies differentiate themselves in the products they offer or should they offer the same set of products?

How does the middle-tier or the competition of the middle-tier firms affect the total market size and the market shares of the producers? Can the producers mitigate or compensate the deleterious effects of the middle-tier firms, if any?

The decisions by the producers and assembly companies define a multiple-leader Stackelberg game. Producers are the leaders. They play a noncooperative game with complete information (including prices and quality levels) with each other. The assembly companies are followers who also play a noncooperative game with complete information with each other. In the leaders’ game, two producers simultaneously decide which product(s) to offer. Then, they set the prices charged to the assembly companies. In the followers’ game, assembly companies, given the transfer price(s) charged by the producers, simultaneously determine which product(s) to offer to the end customers and set the prices for the end customers. As the product variety decisions imply investment in product design and machinery, they precede the product pricing decisions. This is true for both the producers and the assembly companies. In this hierarchical game, we analyze the subgame perfect Nash equilibrium.

Figure 1  Supply chain configurations
In contrast to previous studies we assume that the product variety decisions of the companies in the supply chain are endogenous. That is, each company is capable of manufacturing/selling a set of vertically differentiated products. We also analyze the impact of the middle-tier on the product variety and pricing decisions. We consider not only the effects of the retailers, but also the assembly operations which involve additional bill-of-materials (BOM) cost. BOM includes the raw materials and components needed to manufacture a product as well as the cost of producing, managing and synchronizing supply, inventory, storage and distribution of these materials and components. The relationship of the BOM cost and product quality changes according to the industry. In the semiconductor industry, a high-end chip is usually matched with high-end video cards and high-end memory, so the BOM cost of other components typically increases with the quality of the chip. In the other extreme, for products such as bottled water, the BOM cost of other materials consists mainly of packaging cost, which does not vary much with the water quality (e.g., purified water, mineral water, vitamin water, etc.) and can be viewed as a constant. In this paper, we consider both the case in which the BOM cost is a linear function of the product quality and the case in which the BOM cost of other components is a constant.

Our results for the producers’ product variety decisions are consistent with the literature: Producers should use product differentiation to defend themselves against the competition (Moorthy 1988, Shaked and Sutton 1982). Our insights on the impact of the supply chain configurations and of the BOM costs, however, are new and unexpected:

i) The commonly-held view regarding the middle tier in a supply chain is that it hurts the producer due to double marginalization (Tirole 1988, Villas-Boas 1998). We show in this paper that, although a monopolistic middle tier does increase the final selling price to the consumers and lowers the total market share, the competition in a duopolistic middle tier can mitigate that effect. Since in equilibrium companies would choose to differentiate their product, the degree of such mitigation varies by the quality difference between the low and high-end products. In particular, the smaller the quality difference, the stronger the competition and thus the less severe the effect of double
marginalization. In addition, the effect of the middle tier on the producers is asymmetric: the low-end producer always obtains a higher profit with a duopoly middle-tier than with direct shipping whereas the high-end producer does better than direct shipping only if the quality difference is small enough.

ii) BOM costs have an impact on the equilibrium product variety and pricing decisions of the assembly companies. Specifically, with a constant cost for additional BOM (excluding the component supplied by the producers), a partial product differentiation equilibrium exists for the assembly companies whereby one company produces both high and low quality products and the other one produces only low quality products. The resulting competition in the low quality market segment drives the profit of the assembly companies for the low quality product to zero, effectively eliminating the impact of the middle-tier and mitigating the impact of the high BOM cost. When the BOM cost is very high, the low-end product becomes too expensive relative to its quality level and all companies in the supply chain offer the high-end product only.

iii) The presence of a middle-tier affects the quality level decisions of the producers. Villas-Boas (1998) suggests that a monopolistic company should differentiate the quality levels of its products more when there is a middle-tier, this is not true in the two-producer and two-assembly company setting. Instead, a low-end producer should position its quality level closer to the high-end producers when there is a duopoly in the middle tier.

The organization of the rest of the paper is as follows. In Section 2 we review the related literature. In Section 3 we introduce the customer utility model and the product variety and pricing subgames between the producers and assembly companies. Section 4 includes the analysis of games for different supply chain configurations and different BOM cost structures. We also compare the supply chain configurations and discuss managerial insights in these section. In Section 5 we present extensions of our model. Finally, we conclude and discuss future research in Section 6. The Appendix provides the proofs for all the propositions.
2. Background

Our work is related to three main streams of research concerned with product variety decisions and their influence on product pricing: the effects of vertical differentiation in a monopolistic setting, the effects of vertical differentiation in a competitive environment and the influence of the distribution channel on those decisions.

To the best of our knowledge, Mussa and Rosen (1978) is the first work that considers vertical differentiation. They show that price discrimination is optimal for vertically differentiated products. Moorthy (1984) generalizes the results of Mussa and Rosen (1978) for different customer utility and cost functions. Barghava and Choudhary (2001) show that a monopolistic company's optimal product line depends on the benefit-to-cost ratios of the products. They find that vertical differentiation strategy is not optimal when the highest quality product has the best benefit-to-cost ratio. These studies consider the product variety problem from the manufacturer’s point of view. Recent work of Pan and Honhon (2012) takes the retailer’s point of view and considers a product assortment problem. The authors study the optimal product assortment decisions for both exogenous (given) and endogenous (variable) pricing mechanisms, and introduce an algorithm based on the shortest path problem. They also consider the effect of the fixed costs on product assortment decisions. All these studies address vertical differentiation for a monopolistic company.

Vertical differentiation with competition is considered in Chambers et al. (2006), Jing (2006), Rhee (1996), Moorthy (1988) and Shaked and Sutton (1982). Each company positions itself on a quality level. Then, companies compete on both quality and selling prices. Moorthy (1988) and Shaked and Sutton (1982) conclude that companies should differentiate in the products they offer in a duopolistic setting. Rhee (1996) considers heterogeneity not only in the willingness to pay for quality but also along other unobservable attributes which companies cannot manage strategically. He points out that customer heterogeneity along these unobservable attributes drives companies to offer products of similar quality levels. Desai (2001) also incorporates the taste preferences of the customers and concludes that cannibalization and competition are the two important forces that
drive the companies’ strategies. Jing (2006) focuses on how a company can assess the profitability of a potential brand location in an oligopolistic setting. He has identified the conditions under which it is more profitable to offer a higher- or lower-quality product for a company. He also provides a set of profitability bounds. Chambers et al. (2006) discusses the trade-offs between variable cost and quality in a duopolistic setting, i.e. they show that quality levels offered by the companies depend on the variable cost structure. All these studies take either a producer’s or a retailer’s point of view. Moreover, there consider a single stage supply chain.

A recent paper by Chen et al. (2013) examines an incumbent OEM’s decision to either develop its own low-end version of an existing high-end product or to allow an entrant rival to produce the low-end extension. The OEM and its rivals both interact with a common critical-component supplier and when the parties act strategically, they show that the competition can benefit the incumbent OEM even if it does not collect any licensing fees from the rival. We also model a similar two-stage supply chain in this paper, although we consider a setting with two competing suppliers (i.e., producers) and we allow both the producers and the assembly companies to determine their own product variety.

A stream of research considers the effect of the distribution channels when designing a product line. Villas-Boas (1998) studies the effects of different distribution channels on a manufacturer. They conclude that channel pricing mechanisms increase the cannibalization forces across the product line. According to this work, the best strategy for a producer distributing through a retailer is to increase the quality level differences in the products being supplied. In this way the manufacturer is making major profit on the high-end segment as well as getting some positive profits from the low-end segments. Moreover, the competition between the retailers targeting different customer segments alleviates this impact. Zhao et al. (2009) study the effect of vertical integration on two manufacturers and two retailers. In their work, production and distribution channels for the high-end and low-end products are different, i.e. each manufacturer distributes its product through only one retailer. The asymmetry in the distribution channel also affects the quality decisions of
the companies. The high-end manufacturer prefers vertical integration with the retailer whereas the low-end manufacturer gains more profit with decentralization. Hua et al. (2011) consider a single manufacturer and a single retailer that serve customers in two market segments, i.e. high-end and low-end segments. They study the equilibrium product variety decisions in a Stackelberg game framework (non-cooperative scenario), and compare this non-cooperative scenario with a revenue-sharing contract scenario. They show that revenue-sharing contract significantly improves the supply chain performance. Federgruen and Hu (2012) also study a multi-level supply chain with multiple products where companies in each level engage in a price competition. They assume that the product assortment offered by the suppliers is given, and they characterize the price equilibrium for different supply chain configurations. Shi et al. (2013) analyze a manufacturer’s optimal quality decisions in different distribution channels with different types of customer heterogeneity and different customer distributions. They state that decentralization and centralization together with the customer heterogeneity and customer distributions have an impact on the product quality decisions of the companies.

We integrate these three different research streams by considering vertically and horizontally competing companies that are capable of producing more than one product, i.e. a two-stage supply chain with two products. In addition, the middle-tier companies are not restricted to distribution operations but can also perform assembly and manufacturing operations on the products. As a result, our modeling framework allows us to analyze the product variety problem in a more general and realistic environment for different industries. Our work has three main contributions to the current literature: 1) We introduce a two-stage supply chain framework for vertically differentiated products, with multiple interacting parties. We allow competition at both stages to examine the effects of competition and cannibalization. 2) We analyze the effect of the middle-tier companies on the market equilibrium and product variety decisions of producers. 3) In this product variety and pricing problem, we consider internal costs, competition and external factors (customer behaviors and supply chain) in one framework.
3. Model

We consider two identical producers that can offer two vertically differentiated products, named L and H. Quality levels of L and H are given as \( q_L \) and \( q_H \), respectively. We measure quality levels in dollars and define \( q_i \) as the willingness-to-pay of the customer who (among all potential end customers) values the product the most. We assume a uniform distribution of end customers that are willing to pay between 0 and \( q_i \) for product \( i \). We use a uniformly distributed random variable \( \theta \in [0, 1] \) to sample this distribution denoting a customer’s valuation of quality. For example, a customer with \( \theta = 0.5 \) is willing to pay at most half of \( q_i \) for product \( i \). Hence, \( \theta q_i \) denotes the valuation of the customer for product \( i \). A higher \( \theta \) value indicates that the corresponding customer assigns a higher level of importance to quality, so he/she is willing to pay more for the product.

Following Mussa and Rosen (1978), an end customer who buys product \( i \) from assembly company \( j \) with a price of \( r_{ij} \) has a utility of

\[
U_{ij} = \theta q_i - r_{ij}. \tag{1}
\]

Among all the products in the market a customer buys the product \((i^*, j^*)\) with the highest positive utility, i.e.

\[
(i^*, j^*) = \arg \max \{U_{ij} : U_{ij} \geq 0\}. \tag{2}
\]

We assume that the quality levels of the products are exogenously provided leaving producers and assembly companies to make decisions on product variety and pricing. We model this game as a multi-leader Stackelberg game in which producers correspond to the leaders and assembly companies correspond to the followers. Let \( j \) and \( k \) denote the indices for the assembly companies and producers, respectively. First, each producer decides on the set of products offered to the assembly companies, \( Z_k \), and then sets the transfer prices as \( c_{ik} \). A producer can offer product L or H alone, or both products L and H. Next, each assembly company decides on the set of products offered to the end customers, \( Y_j \), and then sets the prices \( r_{ij} \). Based on the products offered by the producers, each assembly company may choose to offer product L or H alone, or offer both. Each company makes a decision considering the product variety and pricing decisions of its competitors,
so we use the notation $Z_{-k}$ and $Y_{-j}$ for the set of products offered by the producer except producer $k$ and that by the assembly company except assembly company $j$, respectively.

In semiconductor chip production, the CPU speed of a chip is essentially determined by a random yield process and hence the production costs of a low-end and a high-end product are essentially the same. The major costs in semiconductor production are sunken capital and development cost. Hence we ignore variable production costs and assume that each producer maximizes revenue. For industries where such variable costs are not negligible, we extend the model to consider producers’ variable costs in Section 5.

We assume that each assembly company maximizes its profit. The cost of each assembly company varies according to its product variety. It includes two items: the transfer price from the producers $c_{ik}$, and the cost of other components in the BOM (video card, memory, etc). For brevity, we refer to the additional BOM cost simply as the “BOM cost” hereafter. As explained in the Introduction, we consider two types of BOM costs: quality-dependent BOM cost and quality-independent BOM cost. In the former we assume that the BOM cost is linearly increasing with the quality level. We define $b_q$ as the BOM cost for product $i$ where $b$ is the BOM cost for unit quality and without loss of generality, we assume $b < 1$ (note that $b \geq 1$ would imply that the BOM cost is higher than the maximal achievable price for the product at the retail level). Linearly increasing BOM cost are typical in the semiconductor industry: The quality level of a chip determines the quality levels of the other components of the product. When the BOM cost are quality-independent, the dominant parts are overhead costs such as packaging, storage, and transportation. The bottled water industry is a typical case for a constant BOM cost. We define $a$ as the constant BOM cost for each product. In Section 4 we analyze these two types of BOM cost in detail.

Let $P_{ijk}(Y_j, Y_{-j}, Z_k, Z_{-k})$ denote the market share of product $i$ procured from producer $k$ and sold by assembly company $j$. We normalize the total potential market size to 1. We define the market share of a product as the fraction of all potential who buy this product, as in Barghava.
and Choudhary (2001) and Pan and Honhon (2012). The revenue of producer \( k \), \( \Pi^p_k \), depending on the product variety decision made by both producers coded in \((Z_k, Z_{-k})\), is given by

\[
\Pi^p_k(Z_k, Z_{-k}) = \sum_{i \in Z_k} \sum_{j} c_{ik} P_{ijk}(Y_j, Y_{-j}, Z_k, Z_{-k}) \tag{3}
\]

and the profit of assembly company \( j \), \( \Pi^a_j \), becomes

\[
\Pi^a_j(Y_j, Y_{-j}) = \sum_{i \in Y_j} \sum_{k} (r_{ij} - c_{ik} - b_{q_i}) P_{ijk}(Y_j, Y_{-j}, Z_k, Z_{-k}) \tag{4}
\]

\[
\Pi^a_j(Y_j, Y_{-j}) = \sum_{i \in Y_j} \sum_{k} (r_{ij} - c_{ik} - a) P_{ijk}(Y_j, Y_{-j}, Z_k, Z_{-k}) \tag{5}
\]

for constant and quality-dependent BOM cost, respectively.

Pan and Honhon (2012) state conditions for obtaining positive market shares for products sold by a monopolistic company, which apply to the competitive environment as well. In the following proposition, we restate their result in the context of our model.

**Proposition 1.** In a competitive market, products have positive market shares if and only if

\[
0 \leq \frac{r_L}{q_L} < \frac{r_H - r_L}{q_H - q_L} < 1 \quad \text{holds where } q_L < q_H \quad \text{and } r_i = \min_{j \in \{1, 2\}} \{r_{ij}\}, \quad i = L, H.
\]

Defining \( F(\theta) \) to be the cumulative distribution of the customers. Then, the market shares of assembly company \( j \) can be calculated as

\[
\sum_{k} P_{Ljk}(Y_j, Y_{-j}, Z_k, Z_{-k}) = \begin{cases} 
\frac{1}{2} \left( F\left( \frac{r_H - r_L}{q_H - q_L} \right) - F\left( \frac{r_L}{q_L} \right) \right) & \text{if } r_L = r_{Lj} < r_{L,-j}, \\
0 & \text{if } r_L = r_{Lj} = r_{L,-j}, \\
1 - F\left( \frac{r_H - r_L}{q_H - q_L} \right) & \text{if } r_H = r_{Hj} < r_{H,-j}, \\
\frac{1}{2} \left( 1 - F\left( \frac{r_H - r_L}{q_H - q_L} \right) \right) & \text{if } r_H = r_{Hj} = r_{H,-j}, \\
0 & \text{otherwise.}
\end{cases}
\]

\[
\sum_{k} P_{Hjk}(Y_j, Y_{-j}, Z_k, Z_{-k}) = \begin{cases} 
\frac{1}{2} \left( 1 - F\left( \frac{r_H - r_L}{q_H - q_L} \right) \right) & \text{if } r_H = r_{Hj} < r_{H,-j}, \\
1 - F\left( \frac{r_H - r_L}{q_H - q_L} \right) & \text{if } r_H = r_{Hj} = r_{H,-j}, \\
0 & \text{otherwise.}
\end{cases}
\]

We observe two domination cases (in which the market share of the dominated product is zero) in the market. Domination could occur within the same segment, i.e., the low(high)-end product
by one firm is dominated by the low(high)-end product by the other firm, or between different segments, i.e., the low(high)-end product by one firm is dominated by the high(low)-end product by the other firm. Domination within the same segment occurs if there is a cheaper product with the same quality level in the market, i.e. $r_{Hj} < r_{H,-j}$ or $r_{Lj} < r_{L,-j}$; the cheap product dominates the expensive product. This is stated in equations (6) and (7). Domination between segments occurs when the condition in Proposition 1 is violated: 1) If the high-end product has a lower price-to-quality ratio, i.e. $r_{L}q_{H} \geq r_{H}q_{L}$, it dominates the low-end product. 2) If the difference in price of the high and low-end products is higher than the quality difference, i.e. $r_{H} - r_{L} \geq q_{H} - q_{L}$, the low-end product dominates the high-end product. In both 1) and 2) the firm will simply offer the dominating product alone.

4. Analysis of Three Supply Chain Configurations

In order to understand the impact of the middle-tier assembly companies we study the multi-leader Stackelberg games defined in Section 3 for the three different supply chain configurations - direct shipping, monopoly of a single assembly company and duopoly of two assembly companies. We assume a duopoly at the producer level across all three configurations. We use backward induction to analyze the subgame perfect Nash equilibria (NE) for each configuration: We begin by setting the equilibrium prices followed by determining the NE for the product variety decisions for the assembly companies. Based on the game outcome of the assembly companies, we solve the producers’ equilibrium transfer prices to the assembly companies and finally determine which products the producers should offer.

4.1. Quality-dependent BOM cost

First we explore the three supply chain configurations with BOM cost that are linearly increasing with the quality level, $b_{q}$.

4.1.1. Direct shipping For direct shipping, each producer has to incur the BOM cost. Each producer has three options: (1) offer product L only, (2) offer product H only, and (3) offer both
products L and H. For each of these options, the producer determines the selling prices of the products to maximize its profit, i.e.

$$\max_c \Pi_{k}^{p,s}(Z_k, Z_{-k}, c) = \sum_{i \in Z_k} (c_{ik} - bq_i) P_{ik}(Z_k, Z_{-k})$$

(8)

where the superscript $s$ denotes the direct shipping configuration. Since there is no assembly company in the market, selling prices to the end customers become $c_{ik}$. Producers follow a pricing strategy such that neither producer deviates unilaterally from equilibrium prices, resulting in

$$\Pi_{k}^{p,s}(Z_k, Z_{-k}, c^*) \geq \Pi_{k}^{p,s}(Z_k, Z_{-k}, c)$$

for $k = 1, 2$.

**Proposition 2.** There is a unique price and product variety equilibrium for the producers in which the producers differentiate in the products they offer, i.e. one of them offers product H (high-end producer) and the other offers product L (low-end producer). The equilibrium transfer prices for the low-end and high-end producers are $c_{L}^* = \frac{q_L((1 + 3b)q_H - q_L)}{4q_H - q_L}$ and $c_{H}^* = \frac{q_H(2(1 + b)q_H - (2 - b)q_L)}{4q_H - q_L}$, respectively.

In the Nash equilibrium each producer chooses a product different from its competitor’s product. This result is in line with the product differentiation discussed in Moorthy (1988). Product differentiation weakens price competition and increases profits of the producers. Note that BOM cost increases the equilibrium prices of both products.

### 4.1.2. Single assembly company (monopoly)

Monopolistic assembly company manages the BOM. The decision problem of the assembly company becomes

$$\max_r \Pi_{a,m}^{n,m}(Y, r) = \sum_{i \in Y} \sum_{k} (r_i - c_{ik} - bq_i) P_{ik}(Y, Z_k, Z_{-k})$$

(9)

where the superscript $m$ denotes the supply chain setting with a single assembly company.

**Proposition 3.** There is a unique price and product variety equilibrium for the assembly company and producers in which the assembly company offers both products L and H at the equilibrium prices $r_{i}^* = \frac{c_i^* + (1 + b)q_i}{2}$ for $i = L, H$ and the producers differentiate in the products they offer, i.e. one of them offers product H (high-end producer) and the other offers product L (low-end producer) at equilibrium transfer prices $c_{L}^* = \frac{(1 - b)q_L(q_H - q_L)}{4q_H - q_L}$ and $c_{H}^* = \frac{2(1 - b)q_H(q_H - q_L)}{4q_H - q_L}$. 
In this supply chain configuration, both the producers and the assembly company offer vertically differentiated products. Comparing this with Proposition 2, we observe that the transfer prices of the producers is lower than the direct shipping selling price by \( b_q \) (i.e., the BOM cost). Although the transfer prices of the producers differ by \( b_q \) between the monopoly and single assembly cases, the market shares of the products decrease by half. This is due to the double marginalization effect of the middle-tier as indicated in Tirole (1988) and Villas-Boas (1998).

4.1.3. Two assembly companies (duopoly)

**Proposition 4.** In the two-assembly-company configuration, there exist two Nash equilibria (NE) for the subgame between the assembly companies:

- **Product-differentiation** equilibrium. The two assembly companies differentiate in the products they offer, i.e. one of them offers product \( H \) (high-end assembly company) and the other offers product \( L \) (low-end assembly company). This is a strict Pareto efficient NE. The equilibrium prices are
  \[
  r^*_L = \frac{2q_H c^*_L + q_L c^*_H + q_L (q_H - q_L) + 3q_L q_H b}{4q_H - q_L} \quad \text{and} \quad r^*_H = \frac{q_H c^*_L + 2q_H c^*_H + 2q_H (q_H - q_L) + b q_H (2q_H + q_L)}{4q_H - q_L}.
  \]

- **No-product-differentiation** equilibrium. Both assembly companies offer both products \( H \) and \( L \). This is a weak Pareto dominated NE. The equilibrium prices are \( r^*_i = c^*_i + b_q \), for \( i = L, H \).

The producers differentiate in the products they offer, i.e. one of them offers product \( H \) (high-end producer) and the other offers product \( L \) (low-end producer). In addition,

- in the case of the no-product-differentiation equilibrium in the middle-tier, the equilibrium transfer prices for the low-end and high-end producers are \( c^*_L = \frac{(1 - b) q_L (q_H - q_L)}{4q_H - q_L} \) and \( c^*_H = \frac{2(1 - b) q_H (q_H - q_L)}{4q_H - q_L} \), respectively.

- in the case of the product-differentiation equilibrium in the middle-tier, the equilibrium transfer prices for the low-end and high-end producers are \( c^*_L = \frac{2q_L (1 - b)(q_H - q_L)(3q_H - q_L)}{16q^2_H - 17q_H q_L + 4q^2_L} \) and \( c^*_H = \frac{(1 - b) q_H (q_H - q_L)(8q_H - 3q_L)}{16q^2_H - 17q_H q_L + 4q^2_L} \), respectively.

On the one hand, when there is **product-differentiation** in the middle-tier, assembly companies add a profit margin over the profit margin of the producers. The increase in the selling prices
reduces the customer demand. On the other hand, when there is no-product-differentiation in the middle-tier, the competition leads to marginal cost pricing driving the profits of the assembly companies to zero. This eliminates the impact of the middle tier companies and results in the largest total market penetration (i.e., total market share) among all equilibria.

Which equilibrium will be observed in the middle-tier eventually, product-differentiation or no-product-differentiation? Assuming that the assembly companies are rational and that they have complete information of each other’s cost and pricing structure, they will choose the Pareto efficient equilibrium of product-differentiation. The no-product-differentiation equilibrium is a weak NE, which means that each company has an alternate response to the other company’s strategy that leads to the same profit. Hence the weak NE is less stable compared to the strict NE case, and the product-differentiation equilibrium is more likely to be observed in the market.

4.1.4. Comparison of supply chain configurations In all three supply chain configurations, product-differentiation is the dominating equilibrium or Pareto efficient equilibrium outcome for the assembly companies and the producers. For the remainder of the paper, we always refer to the duopoly equilibrium as the one with product differentiation since the no-differentiation equilibrium is a weak equilibrium. We should keep in mind that in duopoly with no-product-differentiation producers have the same market shares and revenues as in the direct shipping. Hence, perfect competition between the assembly companies eliminates the impact of the middle-tier.

Next, we compare the equilibria observed for the three supply chain configurations in terms of prices, market shares, profits, and revenues. We use superscripts $s$, $m$ and $d$ for the direct shipping, monopoly of assembly company and duopoly of assembly companies with product-differentiation, respectively. Recall that at the producers’ level, it is always a duopoly game.

**Proposition 5.** For each product $i$ the equilibrium selling prices $c^*_i$, market shares $P^*_i$, and revenues $(\Pi^*_i)^p$ of the producers are ordered as

\[ (c^*_i)^d > (c^*_i)^s = (c^*_i)^m + b, \quad i = L, H; \]  \hspace{1cm} (10)
\[
\sum_{j} (P_{ij}^*)^s > \sum_{j} (P_{ij}^*)^d > (P_i^*)^m, \quad i = L, H; \quad (11)
\]
\[
(\Pi^*_L)^{p,d} > (\Pi^*_L)^{p,s} > (\Pi^*_L)^{p,m}, \quad (12)
\]
\[
(\Pi^*_H)^{p,s} > (\Pi^*_H)^{p,d} > (\Pi^*_H)^{p,m}, \quad \text{if } q_L \leq 0.69q_H \quad (13)
\]
\[
(\Pi^*_H)^{p,d} > (\Pi^*_H)^{p,s} > (\Pi^*_H)^{p,m}, \quad \text{if } q_L > 0.69q_H \quad (14)
\]
\[
(\Pi^*_L)^{p,t} > (\Pi^*_L)^{p,t}, \quad t = s, m, d. \quad (15)
\]

To facilitate the comparison between the equilibria of the different supply chain configurations we rank them by prices, market shares, profits and revenues in Table 1. In all supply chain configurations, the high-end producer obtains more revenue than the low-end producer. Both high-end and low-end producers obtain the lowest revenue in the monopolistic configuration among all three supply chain configurations. Compared to direct shipping, a monopolistic company in the middle-tier leads to higher selling prices to the consumers. Hence, even though the transfer prices

<table>
<thead>
<tr>
<th>Configuration /Equilibrium</th>
<th>Market price</th>
<th>Transfer price</th>
<th>Market share</th>
<th>Profit (ACs)</th>
<th>Revenue (PRs)</th>
<th>Consumer Surplus (ECs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct shipping</td>
<td>3</td>
<td>3</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>2 1* or 2**</td>
</tr>
<tr>
<td>Product differentiation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single assembly company</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Product differentiation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two assembly companies</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Product differentiation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* if \(q_L \leq 0.69q_H\)

** if \(q_L > 0.69q_H\)
for the producers in the direct shipping and monopoly settings are the same, double marginalization due to the middle tier results in a decrease in sales, leading to a lower revenue for the producers. As a result, direct shipping has the largest total market market share among all configurations.

The low-end producer can charge higher prices in the duopoly than in the other configurations, and it can capture a larger share of the overall market. Hence, the low-end producer always prefers the duopoly configuration. However, the best configuration for the high-end producer depends on the quality level difference between the two products. On the one hand, if the quality levels between products H and L are close, in particular, $q_L > 0.69q_H$, the high-end producer favors the duopoly configuration. In this case products are similar to each other and the competition between the two products is more intense, which dampens the negative effect of double profit margins. On the other hand, the direct shipping configuration is better for the high-end producer when the quality levels are more different, i.e. $q_L \leq 0.69q_H$ because with “more” differentiated products, the competition between the assembly companies is not sufficiently intense to offset the negative effect of double profit margins in a two-stage supply chain.

The revenues of both producers are hurt by the monopolistic middle-tier. This is rather intuitive, as the monopolistic assembly company becomes the most powerful player in the market and it has the control of the market. When there is competition in the middle-tier, power is more distributed leading to interesting results for the producers: The conventional wisdom in supply chain management suggests that the “middle-tier” tends to decrease a producer’s payoff due to double marginalization. However, we find that the low-end producer is always better off when there is competition in middle-tier. The reason is that the cost structure changes when two assembly companies compete (instead of the two producers compete directly as in direct shipping): The assembly company offering product H incurs a higher cost than the one offering product L due to the higher transfer price for product H. This exacerbates the double marginalization impact for product H even more, allowing product L to “steal” some of the product H’s market share. Therefore, the effect of the middle-tier on the producers is asymmetric, leading to asymmetric preferences by the producers.
The supply chain configuration preferences of the assembly companies are quite intuitive. A monopolistic assembly company maximizes its profit by offering both products L and H at the highest prices leading to the lowest market shares. Hence, a monopolistic assembly company has the maximum profit among all supply chain configurations. In the case of duopoly, the company with market segment H gains higher profit than the company in the market segment L.

Next, we examine the impact of the supply chain configuration on consumer surplus. The surplus of an individual customer who has a quality valuation of $\theta$ for a product can be calculated as the net utility gained from buying one unit of a product with quality $q_i$ at price $r_{ij}$, i.e. $s_\theta = \theta q_i - r_{ij}$. If the customer does not buy any product, then the surplus is $s_\theta = 0$. Given an equilibrium point for a supply chain configuration $t$, the customers’ total surplus can be calculated as

$$CS^t = \int_{\theta_1}^{\theta_2} (\theta q_{iL} - (r^*_{iL})) d\theta + \int_{\theta_2}^{1} (\theta q_{iH} - (r^*_{iH})) d\theta. \quad (16)$$

where $\theta_1 = \frac{(r^*_{iL})}{q_{iL}}$ and $\theta_2 = \frac{(r^*_{iH}) - (r^*_{iL})}{q_{iH} - q_{iL}}$. Note that $r^*_i$ is replaced with $c^*_i$ in the direct shipping mode.

**Proposition 6.** The customers’ total surplus for the equilibrium points of all supply chain configurations satisfies

$$CS^s > CS^d > CS^m. \quad (17)$$

Customers prefer a market where there is no middle tier since direct shipping leads to the lowest selling prices. When assembly companies focus on different market segments, i.e. in the duopoly with product-differentiation equilibrium, the competition between the products is reduced leading to higher prices than in the direct shipping case. In the case of a single assembly company, lack of competition in the market causes even higher selling prices, hurting the end customers.

4.1.5. Discussion and Insights Producers should focus on different segments of the market and differentiate in the products they offer, independent of the middle-tier structure. In this way, producers can take advantage of the heterogeneity of the market and avoid the most fierce form of
competition with each other. Nonetheless, the structure of the middle-tier affects market shares, selling prices and revenues of the producers. Particularly, a monopolistic company in the middle-tier is the worst scenario for the producers (and the end customers) as it pushes down the market shares and revenues of the producers to the minimum levels while the customers experience the highest prices.

As long as the quality levels of the products are close to each other, both producers will benefit from the existence of a middle-tier with competition. This is a counterintuitive result as usually a middle-tier leads to higher selling prices which reduces demand due to double marginalization (Villas-Boas 1998, Tirole 1988). However, product differentiation in the producers and the assembly companies compensates for this double marginalization. Hence both producers are better off with a middle tier with competition than with direct shipping or with a monopolistic middle tier.

When the quality levels of the producers differ significantly, the interests of the producers diverge: The low-end producer favors a middle-tier with competition whereas the high-end producer maximizes his revenue via direct shipping. The later is also the preferred configuration for the end customers. The effect of choosing optimal quality levels in this situation is further examined in Section 5.

The current state of the semiconductor industry is similar to the Nash equilibrium in a middle-tier with competition. There are two producers: a highly profitable high-end producer (INTEL) and the less profitable low-end producer (AMD) and two OEM clusters: a low-end OEM cluster including Dell, Acer, etc, and a high-end OEM producer (Apple) with the high-end OEM producer having a much higher profit margin than the low-end OEM cluster (Dediu 2013).

4.2. Quality-independent BOM cost

We now explore the three supply chain configurations when the BOM cost per product, \( a \), is constant.

4.2.1. Direct shipping

**Proposition 7.** There is a unique price and product variety equilibrium for the producers:
• If $\frac{q_L}{2} \leq a < q_L$, then product H dominates product L. Both producers offer product H only. The equilibrium price for product H is $c_H^* = a$.

• If $a < \frac{q_L}{2}$, then producers differentiate in the products they offer, i.e. one of them offers product H and the other offers product L. The equilibrium prices are

$$c_L^* = \frac{q_L(q_H - q_L) + a(2q_H + q_L)}{4q_H - q_L} \quad \text{and} \quad c_H^* = \frac{q_H(2(q_H - q_L) + 3a)}{4q_H - q_L},$$

respectively.

In the first case, the BOM cost for product L is more than 50% of the maximum willingness-to-pay in the market. Hence it is too high relative to its quality level and customers buy product H. In the second case, the BOM for product L is low and the price for product L can be cheap enough so that customers with low quality valuation buy product L. In this case, the market becomes big enough so that producers can split it, reducing the competition and leading to higher revenue.

4.2.2. Single assembly company (monopoly)

**Proposition 8.** There is a unique price and product variety equilibrium for the assembly company and producers:

• If $\frac{q_L}{2} \leq a < q_L$, then product H dominates product L. The assembly company and both producers offer product H only. The equilibrium price for product H is $r_H^* = \frac{c_H^* + q_H + a}{2}$. Both producers offer product H with a transfer price of $c_H^* = 0$.

• If $a < \frac{q_L}{2}$, then the assembly companies offer both products H and L with prices $r_i^* = \frac{c_i^* + q_i + a}{2}$ for $i = L, H$. Producers differentiate in the products they offer, i.e. one of them offers product H and the other offers product L. The equilibrium prices for the low-end and high-end producers are $c_L^* = \frac{(q_L - 2a)(q_H - q_L)}{4q_H - q_L}$ and $c_H^* = \frac{(q_H - q_L)(2q_H - a)}{4q_H - q_L}$, respectively.

It is easy to see, that the transfer prices for this supply chain differ from the transfer prices in the direct shipping configuration by a decrease of $a$, corresponding to the reallocation of the BOM from the producers to the assembly company. Overall, as before, the revenue of the producers is less than in the direct shipping case since the monopolistic assembly company leads to an increase in the selling prices and a decrease in sales.
4.2.3. Two assembly companies (duopoly) Define three critical values of BOM cost, $A_1$, $A_2$ and $A_3$ with $A_1 = \frac{2q_L(3q_H - q_L)}{(8q_H - 3q_L)}$, $A_2 = \frac{3q_L}{4}$, $A_3 = \frac{q_L(-16q_H^2 + 28q_Lq_H^2 - 19q_H^2 - 4q_L^3)}{2(-32q_H^3 + 56q_Lq_H^2 - 33q_H^2q_H + 6q_L^3)}$ where $A_1 > A_2 > \frac{q_L}{2} > A_3$.

**Proposition 9.** There is a unique price and product variety equilibrium for the assembly companies and producers:

- If $A_1 \leq a < q_L$, then product $H$ dominates product $L$. Assembly companies offer product $H$ only. The equilibrium price for product $H$ is $r_H^* = c_H^* + a$. Both producers offer product $H$ only. The equilibrium prices for product $H$ is $c_H^* = 0$.

- If $A_2 < a < A_1$ or $a < A_3$, then the assembly companies differentiate in the products they offer, i.e. one of them offers product $H$ and the other offers product $L$. The selling prices for the low-end and high-end products are $r_L^* = \frac{q_L(q_H - q_L) + q_lc_H^* + 2q_Hc_L^* + a(2q_H + q_L)}{4q_H - q_L}$ and $r_H^* = \frac{2(q_H - q_L) + 2c_H^* + c_L^* + 3a}{4q_H - q_L}$, respectively. The producers differentiate in the products they offer, i.e. one of them offers product $H$ and the other offers product $L$. The equilibrium prices are

$$c_L^* = \frac{(q_H - q_L)(q_L(6q_H - 2q_L) - a(8q_H - 3q_L))}{16q_H^2 - 17q_Lq_H + 4q_L^2}$$

and

$$c_H^* = \frac{(q_H - q_L)(q_H(8q_H - 3q_L) - a(6q_H - 2q_L))}{16q_H^2 - 17q_Lq_H + 4q_L^2}.$$

- If $A_3 \leq a < A_2$, then the assembly companies offer partial product differentiation, i.e. one of them offers product $L$ only and the other offers both products $L$ and $H$. The selling prices for the low-end and high-end products are $r_L^* = c_L^* + a$ and $r_H^* = \frac{q_H - q_L + c_L^* + c_H^* + a}{2}$, respectively. Producers differentiate in the products they offer, i.e. one of them offers product $H$ and the other offers product $L$. The equilibrium prices are $c_L^* = \frac{(q_H - q_L)(3q_L - 4a)}{8q_H - 5q_L}$ and $c_H^* = \frac{(q_H - q_L)(4q_H - q_L - 2a)}{8q_H - 5q_L}$, respectively.

The most interesting case here is the emergence of a new type of Nash equilibria for the assembly companies for $A_3 \leq a < A_2$, a partial-product-differentiation equilibrium, whereby one company offers both products and the other offers only product $L$. 

\[
2(\text{A}_1 - \text{A}_2) > 0.5 \text{A}_1 \left(1 - \frac{3}{2}\text{A}_2 \text{A}_3^{-1}\right) > 0.5 \text{A}_2 \left(1 - \frac{3}{2}\text{A}_1 \text{A}_3^{-1}\right) \]
The quality-independent BOM cost has two important implications: 1) For the very high BOM cost there is a dominating product in the market. There is no benefit for using vertical differentiation, and all the companies in the supply chain should accept zero profit/revenue. 2) The BOM cost strongly influences the product variety decisions of the assembly companies. When there is competition between the companies, the common wisdom is product differentiation. However, our study shows that under high BOM cost assembly companies are better off when there is partial product differentiation. Partial product differentiation imposes intense competition for product L driving down the price to marginal cost, eliminating double marginalization and reducing the impacts of high BOM cost. Partial product differentiation is not an equilibrium for the producers and is only observed in the middle-tier. Why is this the case? Producers can anticipate the decisions of the assembly companies beforehand. Hence, producers can take advantage of the assembly companies to alleviate the impact of the high BOM costs. Particularly, high constant BOM cost affects low-end product adversely, so assembly companies alleviate this effect by decreasing the selling prices of the low-end product to its marginal cost. When BOM costs are linearly increasing with the quality levels, selling prices and transfer prices of both products change proportional to the BOM cost. Hence, quality-dependent BOM cost does not affect product variety and pricing decisions of the companies.

4.2.4. Discussion and Insights We summarize the NE for each supply chain configuration for the quality-independent BOM cost in Table 2. In Figure 2, fixing the quality level of the high-end product at $q_H = 10$, we show the optimal supply chain configuration for the producers depend on the BOM cost and of the quality level of the low-end product. Figures 2.a and 2.b are for the high-end and low-end producers, respectively. Note that Region B is infeasible since the BOM cost is higher than the maximal willingness-to-pay of the consumers for the product.

In general, producers differ in the products they offer except in region “Product H only” where the high-end product dominates the low-end product and hence there is no vertical differentiation. The optimal supply chain configuration for the producers in this case is direct shipping or a duopoly in the middle tier.
For lower BOM costs, the preferences of the low-end and high-end producers differ: The high-end producer prefers either direct shipping or duopoly of assembly companies (depending on BOM and \( q_L \)) whereas the low-end producer always prefers a duopoly of assembly companies. These results are also in line with the quality-dependent BOM cost setting.

Given the best supply chain configuration for the producers, Figure 2.c shows the best product variety decisions for the assembly companies. Assembly companies do not fully differentiate if the BOM cost is high. Instead, they decrease the selling price of the low-end product as much as they can to compensate the high BOM cost. Hence, the quality-dependent and quality-independent BOM costs’ settings can lead to different product variety for the assembly companies.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( b )</th>
<th>Producers</th>
<th>Assembly Company(ies)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Shipping</td>
<td>([q_L, q_L] )</td>
<td>Product H only by both</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>([0, q_L] )</td>
<td>Product differentiation</td>
<td>–</td>
</tr>
<tr>
<td>Single assembly company</td>
<td>([q_L, q_L] )</td>
<td>Product H only by both</td>
<td>Product H only</td>
</tr>
<tr>
<td></td>
<td>([0, q_L] )</td>
<td>Product differentiation</td>
<td>Both products H and L</td>
</tr>
<tr>
<td>Two Assembly Companies</td>
<td>([A_1, q_L] )</td>
<td>Product H only by both</td>
<td>Product H only by both</td>
</tr>
<tr>
<td></td>
<td>([A_2, A_1] )</td>
<td>Product differentiation</td>
<td>Product differentiation</td>
</tr>
<tr>
<td></td>
<td>([A_1, A_2] )</td>
<td>Partial product differentiation</td>
<td>Product differentiation</td>
</tr>
<tr>
<td></td>
<td>([0, q_L] )</td>
<td>Product differentiation</td>
<td>Product differentiation</td>
</tr>
</tbody>
</table>

5. Extensions

In this section we study how variable costs and quality decisions of the producers affect the product variety and pricing decisions for vertical differentiation of products. For tractability we consider the quality-dependent BOM cost as in Section 4.1 throughout the extensions.
5.1. Nonzero variable costs for producers

We have omitted the variable costs of the producers in prior analysis. Here, we generalize our model assuming linear production costs, i.e. producing one product with quality $q_i$ has a cost of $w q_i$. Our cost assumption is also used in Liu and Zhang (2013), and it is a special case of the cost assumptions in Motta (1993). Now each producer maximizes its profit, i.e.

$$
\max_c \Pi_k^{p,t}(Z_k, Z_{-k}, c) = \sum_{i \in Z_k} (c_{ik} - w q_i) P_{ik}(Y, Z_k, Z_{-k}).
$$

(18)
The subgame perfect NE for the assembly companies still holds. Analyzing the producers’ game as in Section 4.1, the resulting equilibrium prices for producers $\hat{c}^*$ are

\[
\begin{align*}
\hat{c}_L^* &= (c_L^*)^d + \frac{q_L(2q_H - q_L)(5q_H - 2q_L)w}{16q_H^2 - 17q_H q_L + 4q_L^2} \\
\hat{c}_H^* &= (c_H^*)^d + \frac{q_H(2q_H - q_L)(4q_H - q_L)w}{16q_H^2 - 17q_H q_L + 4q_L^2} \\
\hat{c}_H^* &= (c_H^*)^m = (c_H^*)^s + \frac{(2q_H + q_L)wq_H}{4q_H - q_L} \\
\hat{c}_L^* &= (c_L^*)^m = (c_L^*)^s + \frac{(3q_Lq_Hw)}{4q_H - q_L}.
\end{align*}
\]

Thus, nonzero variable costs for the producers cause an increase in the equilibrium selling prices for the producers and a decrease in market shares under all supply chain configurations. Nevertheless, the orderings in Proposition 5 continues to hold. However, the set of subgame perfect Nash equilibria changes: For the producers, a no-product-differentiation strategy in which both producers offer both products $H$ and $L$ is also a Nash equilibrium, in addition to the product-differentiation equilibrium. No-product-differentiation is a weak NE, hence it is more likely to observe product-differentiation as the equilibrium for the producers.

5.2. The effect of the quality levels

We assumed that producers offer discrete quality levels that are determined exogenously. In some production scenarios product quality is also determined by a competitive game between the producers, influenced by the expected choices that the assembly companies are making. Next, we investigate how the quality levels of the products affect the selling prices, market shares and profits.

Let $\delta$ denote the difference between the quality levels of high-end and low-end products, i.e. $\delta = q_H - q_L$. We rewrite the equilibrium prices and market shares as a function of $q_L$ and $\delta$.

**Proposition 10.** For all supply chain configurations, given $q_L$, as $\delta$ decreases,

- producers’ and assembly companies’ selling prices for both products $H$ and $L$ decrease.
- producers’ and assembly companies’ market shares for both products $H$ and $L$ increase.
- the total market covered increases.
• In general the revenues of producers decreases. However, the revenue of the low-end producer increases if $q_L > 0.72\delta (q_L > 0.42q_H)$ in the product differentiation setting.

• the profits of the assembly companies decrease.

Intuitively speaking, as the gap in quality shrinks, i.e., $\delta$ becomes smaller, we have two products with similar characteristics competing against each other, which drives the selling prices down. This causes an increase in the market shares of both products, and consequently an increase in the total market. However, the increase in the market shares does not compensate the decrease in selling prices, so the profits and revenues decrease.

Notice the counterintuitive result for the low-end producer: When the difference between the quality levels shrinks, the revenue of the low-end producer increases. This is explained by the fact that the middle-tier and competition together worsen the sales of the high-end product, i.e. double marginalization in the middle-tier increases the selling price of high-end even more compared to the low-end product’s. Hence the low-end producer attracts some of the high-end customers.

Hence, a high-end producer should differentiate as much as possible so that each producer focuses on different segments of the market. This observation also strengthens the importance of the product differentiation strategy for the producers.

5.3. Optimization of the quality levels

We showed in Section 5.2 how quality level of the products affect the revenues and profits. So far, quality levels were given exogenously and production costs did not depend on the quality levels. Now we extend our equilibrium analysis of the multi-leader Stackelberg game to include the cost difference of producing high and low-end products letting the producer decide on optimal quality levels (in addition to prices) of the products that they offer.

Technological improvements and production capabilities affect the quality decisions of a producer. Particularly, in semiconductor industry, the features of a chip generation is a permanent decision due to high capital and development costs. So, we assume that quality decisions precede the product variety and pricing decisions.
Hence the order of the decisions become: First, each producer decides on the quality levels of the products offered to the assembly companies, \( q_i \). Next, each producer determines the set of products offered to the assembly companies, \( Z_k \), and finally sets the transfer prices as \( c_{ik} \). Then, each assembly company decides on the set of products offered to the end customers, \( Y_j \), and then sets the prices \( r_{ij} \).

We assume that producing one unit of product with quality \( q_i \) has a cost of \( wq_i \). Due to technological constraints there are upper and lower limits on the quality levels that can be offered by the producers. We denote them as \( q_{\text{max}} \) and \( q_{\text{min}} \), respectively. The revised model is the same as equation (18). We still work with the assumption that there are two vertically differentiated products, i.e., products L and H, in the market. Thus, a producer can offer only product L or H, or both products L and H. Since the two companies are symmetric ex ante, we focus on cases where \( q_H \) (and \( q_L \)) for both companies are the same. (Strictly speaking, an equilibrium with four different quality levels, i.e., two different \( q_H \) values and two different \( q_L \) values, might be possible, but it would significantly complicate the follow-on analysis of product variety and pricing decisions; thus we do not consider it here.)

We optimize each producer’s profit. The equilibrium quality levels \( q^* \) are determined such that neither producer deviates unilaterally from equilibrium quality levels, resulting in

\[
\Pi_p^{k,t}(Z_k, Z_{-k}, q^*) \geq \Pi_p^{k,t}(Z_k, Z_{-k}, q) \quad \text{for} \quad k = 1, 2 \quad \text{and} \quad t = s, m, d.
\]

**Proposition 11.** If the assembly companies engage in a duopoly with product-differentiation, then the producers offer products with quality levels, \( (q^*_H)^d = q_{\text{max}} \) and \( (q^*_L)^d = 0.74q_{\text{max}} \). Otherwise, the producers offer products with quality levels, \( (q^*_H)^s = (q^*_H)^m = q_{\text{max}} \) and \( (q^*_L)^s = (q^*_L)^m = 0.57q_{\text{max}} \).

The equilibrium result in Proposition 11 is surprisingly simple. In all supply chain configurations the high-end producer always offers a product with the maximum quality level so that entry of another producer to the high-end market segment is prevented. If the two assembly companies focus on different segments of the market, then it is optimal for the producer offering the low-end
product to set its quality level at 74% of the maximum quality; if there is no middle-tier assembly company, or there is a single assembly company, or the two assembly companies do not differentiate and both offer the high and low-end products, then the producer offering the low-end product should set its quality level at 57% of the maximum quality.

To explain the gap between 74% and 57%, we note that when the two assembly companies differentiate their offering, the producer as well as the assembly company offering product L, are not concerned about cannibalization, and thus are more aggressive and position $q_L$ close to $q_H$ (in order to increase market share and maximize revenue). When there is a single assembly company, this assembly company is concerned about cannibalization. Hence, the producers tend to differentiate in the quality levels of the products. When there is no assembly company, producers do not face the adverse effects of double marginalization. Therefore, they position themselves far apart.

Lastly, in the quality equilibrium, Proposition 5 except the ordering for the high-end producer (inequalities 13 and 14) still hold. Direct shipping configuration provides the maximum profit for the high-end producer. Next, duopoly with product differentiation and monopolistic assembly company configurations are profitable for the high-end producer. As a result, supply chain configuration preferences for the low- and high-end producers in the quality equilibrium diverge.

6. Conclusion
In this work we study the product variety and pricing decisions for a duopoly of producers who are planning to reach the end customers through a middle-tier. We assume that the market is potentially served by two products L and H, and all of the producers and assembly companies are capable of producing the two products at the same costs. We compare three supply chain configurations: direct shipping, a monopoly with a single assembly company and a duopoly of two assembly companies. We characterize the production processes in various industries using quality-dependent and quality-independent BOM costs in the assembly stage. The resulting game-theoretic setup is a multi-leader Stackelberg game.

We find that unless the quality-independent BOM cost is high, both horizontal and vertical competition in a supply chain lead producers to differentiate the products they offer. However, total
market shares, selling prices, and revenue figures change for different supply chain configurations. Another result pertaining to quality-independent BOM cost, is that the low-end product is driven out of the market, leaving the high-end product to dominate, if the BOM cost is higher than 50% of the maximal selling price of the product.

A monopolistic middle-tier company reduces the total market and generates the minimum revenues for the producers. A counterintuitive managerial insight is that the low-end producer may profit from the existence of a middle tier. If two assembly companies differentiate in the products they offer, their reduced competition protects the low-end producer from the competition of the high-end producer. The most profitable configuration for the high-end producer depends on the quality level difference between the two products. When the quality difference is high, the high-end producer should eliminate the impact of the middle-tier using direct shipping (or encouraging intense competition between the assembly companies). When the quality difference is low, the high-end producer should work with two assembly companies that differentiate in the products they offer.

In industries with quality-independent BOM costs the product variety decisions crucially depend on the magnitude of the BOM costs. This is especially true for the assembly companies since the equilibrium product variety decisions for them do not always lead to product differentiation. In particular, high BOM costs force both assembly companies to offer the low-end product at marginal cost with only one of the assembly companies offering the high-end product. In this case, the double marginalization impact of the middle-tier decreases to zero for the low-end product and the middle-tier even serves to mitigate the impact of the high BOM costs.

Decisions on the optimal quality levels are also affected by the middle-tier structure. Although Villas-Boas (1998) suggests increasing quality differentiation when there is a middle-tier, the influence of the middle-tier disappears when the assembly companies protect themselves from the intense competition using product differentiation. Thus the competition between the producers drives the quality levels of the products close to each other such that the optimal quality level for the low quality product is about 74% of the high quality product. In all other cases, the quality levels between the high- and low-end products should be more apart.
References


