Design of Robust Distribution Networks Run by fourth Party Logistics Service Providers

M.P.M. Hendriks, D. Armbruster†∗, M. Laumanns‡, E. Lefeber∗, J.T. Udding∗

Abstract

We consider a fourth party logistics service provider (LSP), who faces the problem of distributing different products from suppliers to consumers having no control on supply and demand. In a fourth party set-up, the operations of transport and storage are run as a black box for a fixed price. Within certain boundaries with respect to variability in supply and demand, the LSP is bound to meet certain performance thresholds as determined in a Service Level Agreement (SLA). In this case, the incentive for an LSP is to reduce its operational costs, within the limits set by the SLA. The objective is to find an efficient network topology on a tactical level, which still satisfies the service level agreements on the operational level. We develop an optimization method, which constructs a tactical network topology based on the operational decisions resulting from a given MPC policy. Experiments suggest that such a topology typically requires only a small fraction of all possible links. As expected, the found topology is sensitive to changes in supply and demand averages. Interestingly, the found topology appears to be robust to changes in second order moments of supply and demand distributions.

1 Introduction

The logistics networks considered in this paper consist of production facilities, warehouses and consumers, which are geographically connected by links, e.g. roads, railways, waterways. In long-distance transportation networks, the distribution of products is often performed by a logistics service provider (LSP). As opposed to the common supply chain studies, the problem addressed here is not to control the amount of products in the supply chain. Instead, this study addresses one of the typical services an LSP provides: supply and demand cannot be influenced by the LSP, but are simply a (stochastic) reality, that is revealed only a few days in advance. A supplier pushes its products from several production facilities to the logistics provider. By the same token, consumers pull products from the network. The logistics provider then has to decide whether to store the products in a warehouse or to immediately match them with a consumer demand.

The decisions regarding design and operation of distribution networks can typically be classified into three levels: The strategic level, the tactical level and the operational level. The strategic level deals with decisions regarding the number, location and capacities of warehouses. These decisions have a long-lasting effect on the system’s performance. The tactical level includes decisions on which line hauls (transportation link between two facilities) to actually establish, i.e. the design of the network topology. A line haul cannot be established and removed on a short term time scale, since it has a large impact on the operations and schedules of both involved facilities. To preserve continuity for employees and to restrict the complexity of organizational tasks, such a network topology should not change too frequently. According to the

*Department of Mechanical Engineering, Eindhoven University of Technology, PO box 513, 5600MB Eindhoven, Netherlands. Corresponding author: Erjen Lefeber (a.a.j.lefeber@tue.nl).
†Department of Mathematics and Statistics, Arizona State University, Tempe, AZ 85287, USA
‡Institute for Operations Research, ETH Zurich, 8092 Zurich, Switzerland
economy of scales principle, a line hauls efficiency increases if more products flow through it. The average line haul utilization can simply be increased by decreasing the number of line hauls in the network topology. Therefore, an LSP strives for a network topology with a small number of links. Summarizing, the goal on the tactical level is to construct a network topology with a small number of fixed line hauls. Decisions on the network topology are reconsidered on the medium term time scale. The operational level refers to short term decisions such as scheduling and routing of the daily shipments given the tactical topology.

In this paper, we focus on the interplay between the operational level and the tactical level, i.e. we are interested in determining the topology of the network for a given operational strategy and given operational parameters (see (Hendriks 2009)). We specifically strive for a robust topology, which can be established for a relatively long period of time (months or years) and is still cost-effective when operational parameters (supply and demand distributions) change.

This research was supported by the LSP “Koninklijke Frans Maas Groep” (in the meantime bought out by DSV), one of the leading logistics service providers in Europe. One of the typical services this LSP provides can be illustrated by the following scenario: a big plastics company produces different types of products at various production facilities throughout Europe. These products are needed by all kinds of industries, mostly automotive industries, at various plants throughout Europe. Average consumption of each type of product by each consumer is known, but the daily demands fluctuate. On the other hand, due to production in batches, machine breakdown etc., also the supply of the different types of products fluctuates on a daily basis. Regardless of this, the consumers still expect, dependent on the type of product, a certain level of in-time delivery on a daily basis, where it does not matter from which production facility a consumer receives its products. Neither the consumers nor the supplier want to deal with the logistics of buffering the variability of supplies and demands and decide to hand this task to an LSP.

The LSP, in this case, acts as a so-called fourth party logistics services provider. In a costs+ operation, where the customer is charged the actual costs of running the logistics operation plus some margin, the incentive for the LSP to reduce its operational costs is minimal. They are charged to the customer anyway. In a fourth party set-up, however, the operation is run as a black box for a fixed price. Within certain boundaries with respect to variability in supply and demand, the LSP is bound to meet certain performance thresholds. The (two-sided) contract establishing such an agreement is called a Service Level Agreement (SLA). In this case, the incentive for an LSP to reduce its operational costs, within the limits set by the SLA, obviously is much bigger.

Within the bounds set by the SLA, it is the task of the LSP to supply the desired quantities at the desired day. The LSP can compensate for the stochasticity in supplies and demands by temporarily storing products in warehouses rather than shipping them directly from supplier to consumer. Furthermore, shipping via a warehouse enables product mixing so as to leverage on the economy of scales. On the other hand shipping through a warehouse introduces additional delay due to the handling activities, e.g. (un)loading and consolidation. In settings like the automotive industry, actual orders are accurately released to the LSP a couple of days in advance. The LSP can use this information to decide which transportation links to use on the long run and how much of which product(s) to ship through them each day, such that total costs, including transportation and storage costs and penalties for early and late deliveries, are minimized.

In this paper, we address the above described problem and determine the structure of the tactical network topology (the link or line hauls to choose) dependent on the decisions constructed at the operational level. We specifically strive for a network with a very small number of links that still has close to minimal operational cost. Reducing the number of links is a surrogate for a more detailed optimization at the tactical level for which the associated costs (for establishing a link) are hard to quantify:

• A reduction in the number of links reduces the complexity of the network and with that
the complexity of the organizational tasks of an LSP,

- Each link involves a fixed cost due to contracts and overhead,
- Reducing the number of links leads to thicker flows per link.

The overall problem can thus be characterized as a bi-level joint problem: the upper (tactical) level chooses the links and the lower (operational) level, for given choice of links, chooses the material flow. Our goal is to provide an efficient method to approximate the trade-off frontier of link cost versus operational cost for a given time series of supply and demand, and in particular to determine a network topology that is (i) approximately optimal in both objectives (small number of links and minimal operational costs) and (ii) robust with respect to stochasticity (second order moments) of supply and demand. An extensive amount of experiments is performed to confirm these properties.

1.1 Related work

In (Meepetchdee & Shah 2007), (Cordeau, Laporte & Pasin 2006), (Daskin 1995), (Jaillet, Song & Yu 1996), (Leung, Magnanti & Singhal 1990), (Wasner & Zapfel 2004), (Zapfel & Wasner 2002), (Bertazzi, Paletta & Speranza 2002) networks for different applications are designed based on demand and supply. These studies all assume the deterministic problems (i.e. supply and demand are constant). For instance in (Meepetchdee & Shah 2007), properties of logistics networks structures are determined by optimizing the structural design on a strategic level: given the location of a plant producing one type of product and given the number and location of the consumers, who have to be satisfied, strategic decisions on the number and locations of warehouses are made. In (Meepetchdee & Shah 2007) robustness is the extent to which the system is able to carry out its functions despite some damage done to it, such as the removal of some of the nodes and/or edges in the network. Results suggest that if the maximal robustness level is increased, more warehouses are present in the optimal network, decreasing the network efficiency and increasing its complexity. In this paper, however, we consider the tactical problem of designing a network topology, where the number and locations of nodes are given. Furthermore, we consider the distribution of multiple products and take stochastic supply and demand into consideration while designing a network topology for an LSP. Our definition of robustness is then to which extent the network topology is able to minimize operational costs despite some changes in supply and demand distributions.

A common setting for stochastic supply chain problems can be found in (Tsiakis, Shah & Pantelides 2001), (Kwon, Park, Lee, Kim, Kim & Liang 2006), (Hax & Candea 1984) and (Simchi-Levi, Chen & Bramel 2005). These studies investigate different policies for supply chain management: orders are placed to manufacturing facilities to keep inventory positions in warehouses at a desired level and satisfy consumer demand. Only a restricted number ((Hax & Candea 1984), (Simchi-Levi et al. 2005)) model transportation costs by a stepwise cost-function dependent on the capacity of a transportation device (economy of scales). Such an approach reflects practice more accurately than a linear model at the expense of higher computational time. In this paper, a heuristic is proposed that generates approximately the same network topologies for both the stepwise and the linear model for medium size problems. For large network sizes, which become intractable with the stepwise model, we have to rely on an approach with linear transportation costs. Again, we have to stress that the decision space in a supply chain is crucially different from that of an LSP. Namely, in a supply chain, each member can place orders to one upstream. For an LSP however, both supply and demand are given and cannot be influenced. Despite this difference, some of the supply chain studies mentioned above are related to ours as they use similar approaches on a tactical and/or strategic level.

In (Tsiakis et al. 2001) for instance, the strategic design of a multi-echelon, multi-product supply chain network under demand uncertainty is studied. Given demand, decisions have
to be made on the production amount at each supply facility. The objective is to minimize total costs taking infrastructure as well as operational costs into consideration. The authors consider a steady-state form of this problem and thus all flows between nodes are considered to be time-averaged quantities. A set of (only) three demand scenarios is generated and the objective function is expanded by adding weighted costs for meeting demand in each of these three scenarios. In this paper, on the other hand, the amount of supply at the production facilities is given as well as the consumers’ demands and cannot be controlled. Moreover, we do consider time-variant supply and demand of multiple products and use a rolling horizon approach to decide on the product flows on an operational level. Dependent on the link usage during a large number of time steps in this rolling horizon approach, a close to minimal number of links is established to construct a close to optimal topology. This topology turns out to be robust to changes in second order moments of the supply and demand distributions.

In (Kwon et al. 2006) the number and locations of transshipment hubs (strategic level) in a supply network is determined dependent on the product flows. The authors start from a network in which all potential hubs are present and minimize the costs for the operational activities in the network. Then the decrease in costs is evaluated for each of the cases where one hub is deleted from the network (establishing a hub introduces fixed costs). Next, the hub that causes the largest decrease is deleted from the network. This process is repeated and terminates whenever deleting a hub does not significantly affect the costs. In this paper, a similar approach on a tactical level is applied: we start from a fully connected network, run the operational decision making policy for a certain time period (100 time steps) and in the end delete a least used link over this period. This procedure is repeated until the network is minimally connected. Additionally, experiments suggest that this heuristic generates a topology with a small number of links that is robust to changes in supply and demand distributions.

To our knowledge, only a restricted number of studies consider the viewpoint of an LSP where supply and demand are both stochastic and uncontrollable ((Cheung & Powell 1996) and (Topaloglu 2005)). In (Cheung & Powell 1996), models and algorithms for a one product, multi-stage stochastic distribution problem with recourse are developed. In the first stage, the product flows from plants to the different warehouses have to be decided on, without knowing the demand. Then in the second stage, the demand becomes available and products have to be shipped from the warehouses to the consumers. In this paper, we consider the distribution of multiple products by an LSP and investigate the robustness of generated network topologies. Since supply and demand over a restricted time horizon in the future are known, we can apply a rolling horizon approach rather than a multi-stage approach. As we noticed from practice, such a rolling horizon approach is often used by an LSP.

The authors in (Topaloglu 2005) consider a company, which owns several production plants and has to distribute only one type of product to different regional markets. In each time period, a random (uncontrollable) amount of product becomes available at each of these plants. Before the random demand becomes available, the company has to decide which proportion of the products should be shipped directly and which proportion should be held at the production plants. Linear costs are assigned to transportation, holding and backlog. A look ahead mechanism is introduced by using approximations of the value function and improving these approximations using samples of the random quantities. It is numerically shown that this dynamic programming method yields high quality solutions. One of the results of (Topaloglu 2005) is that most improvements on the operational costs are made when a part of the consumers is served by two plants, rather than one. The study in (Topaloglu 2005) investigates the distribution of only one type of product for only one network configuration with linear transportation costs.

In this paper, on the other hand, we consider the distribution of multiple products and start from considering a stepwise cost function to represent the economy of scales principle. Hence, our methodology takes the possible consolidation of high and low value products to ship full trucks into consideration. Another difference is that in the setting we consider, the exact
supplies and demands over a restricted future horizon are known and hence we can apply a rolling horizon approach. This is another major difference with the study in (Topaloglu 2005), where decisions on how much to send or store have to be made before the actual demand becomes available (recourse model). Furthermore, we expand our study by investigating the robustness of the constructed network topologies, i.e., the dependency of the operational performance of the constructed topologies on supply and demand distributions.

We apply the proposed bi-level optimization method to several real-life networks. The results suggest that in each considered case a distribution network with only a few links provides a large portion of the operational efficiency of the fully connected network. These results confirm the findings of other studies on different kinds of two-echelon networks. Results in (Jordan & Graves 1995) for instance suggest that, for a small theoretical problem, limited flexibility in manufacturing processes (i.e., each plant builds only a few products) yields most of the benefits of total flexibility (i.e., each plant builds all products). However, the authors mention that for more realistic cases they have no guidelines or general approach to add flexibility in an arbitrary network. In this paper, this issue is addressed by developing a heuristic that for more realistic cases and for more than a two echelon system generates a network with a small number of links that still performs well. Moreover, the study in (Jordan & Graves 1995), does not investigate the robustness of the constructed networks to stochastic supply and demand. In this paper, besides proposing a generic design methodology, we also investigate robustness properties of the designed topologies.

1.2 Contributions

We proceed in Section 2 by first addressing the lower-level operational problem (Section 2.1) of controlling the material flow for a given network topology. A stepwise cost function is introduced in the flow model to represent the costs for transportation by trucks (economies of scale). The amount of a supply or demand is defined in space units, i.e., one unit of product occupies a certain space in a truck or a warehouse. Storage costs per unit per day are (therefore) the same for each product type. Costs for early and late delivery do depend on the product type. A model predictive control with rolling horizon (MPC) (Garcia, Prett & Morari 1989) is used, which determines a sub-optimal operational routing schedule for a particular topology and particular supply and demand realizations over a certain time period (in this case 100 time steps). This amounts to solving a sequence of dependent non-linear minimum-cost network flow problems.

In Section 2.2, the upper-level is addressed, which in turn refers to the tactical decision making. We present a branch-and-bound method that computes the operational costs as a function of the number of links for given multiple-product supply and demand time series. As the method is impractical for larger instances, we propose a heuristic that works by iteratively dropping least used links (after determining a routing schedule for a certain time period) to generate an approximation of the costs curve. The heuristic is accurate and much faster than the branch and bound, however still large real-life networks instances are intractable due to the model’s non-linearity. Hence, we replace the stepwise cost model by a linear cost model. Since in the heuristic the link usage depends on the product type(s) through it (the higher the demand for just-in-time delivery, the higher the weight factor), links with products that do not require just-in-time delivery will be dropped first. In the end, these products are then shipped through another link together with products that do require just-in-time delivery. In this way, different product types are consolidated in the linear approach as well. We compare the results to those obtained with the stepwise cost model and find similar results in both the costs curve and the topology structure. For the heuristic approximation, which is very efficient, we can usually provide a network with a small number of links and close to minimal operational costs. From that we can conclude that the topology is also close to optimal.

Results of the branch and bound method and the heuristics approach for small problems are compared in Section 3. Next, the results for the heuristics applied to the stepwise and the linear
model for medium size problems are compared. Moreover, results of the heuristics approach applied to the linear model for several large network configurations are shown in Section 3. These results suggest the validity of the proposed approximative bi-level optimization. Finally, we perform a large amount of experiments to formulate generic insights in the robustness of the heuristic network topology. Since this heuristic network topology results from a particular supply and demand time series, we are interested in the performance of this topology for different time series. Interestingly, the experimental results suggest that the heuristic network topology is only sensitive to the first moment of supply and demand distributions. That is, as long as the heuristic network topology is run operationally for a different supply and demand time series with the same means, the operational costs are still close to minimal (relative to the operational costs in the fully connected network). However, if the means are changed, the operational costs grow large (relative to the operational costs in the fully connected network). Having reasonable forecasts about the individual means of supply and demand we would a priori be able to construct a cost-effective network topology, robust to any time series with these means. We end with conclusions and recommendations for future work in Section 4.

Figure 1: Illustration of the considered logistics networks. An arrow represents a line haul (link) from one facility to another.

2 Bi-level network design problem

We consider the distribution of \( K \) types of products by trucks through a network with \( S \) production facilities, \( W \) warehouses, and \( D \) consumers. Unless stated differently, we use the indices \( s \in \{1, ..., S\} \) for the production facilities, \( w \in \{1, ..., W\} \) for the warehouses, \( d \in \{1, ..., D\} \) for the consumers, and \( k \in \{1, ..., K\} \), for the product types. Products can be sent either directly from supplier(s) to consumer(s) or indirectly via a warehouse, assuming that the LSP is free to choose from which supplier(s) a consumer receives a certain product. Shipments from one warehouse to another are not taken into account yet, but could easily be incorporated into this framework as well. An example network for this type of three-echelon multi-item distribution system is depicted in Figure 1. Even though each link in the graph has a certain length representing the geographical distance between the corresponding nodes, Figure 1 does not show the spatial positioning of the nodes for ease of presentation.

Each production facility supplies one or multiple product types, and each consumer demands one or multiple product types, at each time step (day) \( t \in \{1, ..., T\} \). The supply and demand time series \( S_{ik}(t) \) (supply of product type \( k \) by supplier \( i \) at day \( t \)) and \( D_{jk}(t) \) (demand of product type \( k \) by consumer \( j \) at day \( t \)) are randomly distributed according to a joint distribution that
is defined by the following sampling procedure: for each combination of product type, supplier and consumer, we assume a uniform distribution on an interval \([\mu^k(t) - \sigma^k(t), \mu^k(t) + \sigma^k(t)]\) with given mean \(\mu^k(t)\) and half-length \(\sigma^k(t)\). Next, \(T\) preliminary samples out of each of the \(K \cdot (S + D)\) distributions are taken. The final samples are then determined by adding a constant either to each supply or each demand sample for each product type to enforce that for each product type, the sum of \(T\) supplies equals the sum of \(T\) demands over the horizon.

On a given network, the operational task of the LSP is to decide on the amount of products to be transported over each link, which incurs transportation cost on the links, storage cost in the warehouses, and penalty costs for early or late delivery at the consumers. On the tactical level, the task of the LSP is to choose a subset of links such that it can carry out its operational task well, e.g., with minimal expected costs. Of course, there is an immediate trade-off between the number of links and the operational costs, as additional links can only improve the operational costs due to the added flexibility. In the following, we present an approach to approximate the network structure with a close to minimal number of links and close to minimal operational costs.

### 2.1 Operational decision making

#### 2.1.1 Variables

\[
x^k_{ij}(t) = \text{Amount of products of type } k \text{ transported from node } i \text{ to node } j \text{ at time } t
\]

\[
y^k_w(t) = \text{Amount of products of type } k \text{ in warehouse } w \text{ at time } t
\]

\[
b^k_j(t) = \text{Amount of backlog of product } k \text{ at consumer } j \text{ at time } t
\]

To formalize the operational task of the LSP, we define the operational decision variables \(x^k_{ij}(t) \geq 0\) as the amount of products of type \(k\) transported from node \(i\) to node \(j\) on time step \(t\), where \((i, j) \in \mathcal{U}\), the set of links in the given network. On the supplier’s side, we require that all available products \(S^k_i(t)\) have to be picked up at time \(t\) such that

\[
S^k_i(t) = \sum_{j \in P_i} x^k_{ij}(t) \quad \forall k, i, t \tag{1}
\]

where the set \(P_i\) contains the indices of all the nodes to which production facility \(i\) is connected. On the consumers side, both early and late deliveries might occur and have to be accounted for. For this we introduce state variables \(b^k_j(t)\) as the backlog of product type \(k\) for consumer \(j\) on day \(t\), whose dynamics is

\[
b^k_j(t) = b^k_j(k - 1) + D^k_j(t) - \sum_{i \in Q_j} x^k_{ij}(t) \quad \forall k, j, t \tag{2}
\]

where the set \(Q_j\) contains the indices of all the nodes connected to consumer \(j\). In the warehouses, several processes (e.g. inbound, consolidation) between arriving at and storing in a warehouse cause delay. The total delay is captured in a constant \(\tau_w \in \mathbb{N}, \tau_w < \leq K\), so that products arriving in warehouse \(w\) at time \(t\) can be shipped out \(\tau_w\) time steps later at the earliest. To model this time delay, we introduce additional state variables \(y^k_w(t)\) as the inventory of product \(k\) in warehouse \(w\) on day \(t\), whose dynamics is

\[
y^k_w(t) = y^k_w(k - 1) + \sum_{i \in I_w} x^k_{iw}(k - \tau_w) - \sum_{j \in O_w} x^k_{wj}(t) \quad \forall k, w, t \tag{3}
\]

where the set \(I_w\) contains the indices of all nodes that are sources of warehouse \(w\) and the set \(O_w\) contains the indices of all nodes that are destinations of warehouse \(w\).
The operational cost for time step $t$ is the sum of transportation cost, storage cost, and backlog cost. It can be expressed as a function of the decision variables $x_{ij}^k(t)$ and state variables $b_j^k(t)$ and $y_w^k(t)$ at time $t$ as

$$
\alpha(x_{ij}^k(t), b_j^k(t), y_w^k(t)) = \sum_{(i,j) \in \mathcal{U}} c_{ij} \phi(\sum_{k=1}^{K} x_{ij}^k(t)) + \sum_{k=1}^{K} \sum_{j=1}^{D} \beta^k b_j^k(t)^2 + \sum_{w \in \mathcal{W}} h_w \sum_{k=1}^{K} y_w^k(t)
$$

(4)

where $c_{ij}$ represents the geographical distance between nodes $i$ and $j$, $h_w$ represents the inventory holding costs in warehouse $w$, and $\beta^k$ represents the “value” of product $k$. The reason for not using the absolute value $|b_j^k(t)|$ as a penalty for early or late delivery is that in case of shortages, the quadratic function favors equal distribution of shipments among consumers rather than shipping everything to one consumer. The function $\phi(\cdot)$ expresses quantity-dependent transportation costs. We consider two cases, (i) $\phi(x) \equiv V \frac{x}{T}$ to express stepwise-constant transportation costs due to the use of trucks with unit capacity $V$, and (ii) the identity $\phi(x) \equiv V x$ to express the simplest case of linear transportation costs.

We further assume that suppliers and consumers commit to their real supplies and demands to a constant number of time steps $\Omega$ in advance, where $\Omega \ll K$. Thus, the exact supplies and demands for the current and the next $\Omega - 1$ time steps are known and can be taken into account when deciding on the transportation quantities.

The applied method by Frans Maas is to use a limited look-ahead scheme in a rolling horizon fashion, also called model predictive control (MPC). The basic idea of MPC is that optimization is only performed over a limited horizon where more information about uncertain data is available, or even well representable by deterministic or nominal values, and to choose some reasonable approximation of the value function for the time steps outside the horizon. Here, we use a horizon of $\Omega$ steps as the exact supply and demand data is known over this time frame. Thus, we obtain an approximate control policy $\hat{\mu}$ by solving the following time-expanded network flow problem:

$$
\text{minimize} \sum_{q=t}^{t+\Omega-1} \left[ \sum_{(i,j) \in \mathcal{U}} c_{ij} u_{ij}(q) + \sum_{k=1}^{K} \sum_{j=1}^{D} \beta^k b_j^k(q)^2 + \sum_{w \in \mathcal{W}} h_w \sum_{k=1}^{K} y_w^k(q) \right]
$$

subject to

$$
\begin{align*}
S^k(q) &= \sum_{j \in P_i} x_{ij}^k(q) \\
y_w^k(q) &= y_w^k(q-1) + \sum_{i \in I_w} x_{iw}^k(q - \tau_w) - \sum_{j \in O_w} x_{wj}^k(q) \quad \forall k, i, q \\
b_j^k(q) &= b_j^k(q-1) + D_j^k(q) - \sum_{i \in Q_j} x_{ij}^k(q) \quad \forall k, j, q \\
V \cdot u_{ij}(q) &\geq \sum_{k=1}^{K} x_{ij}^k(q) \quad \forall i, j, q \\
x_{ij}^k(q) &\geq 0 \quad \forall k, i, j, q \\
y_w^k(q) &\geq 0 \quad \forall k, w, q
\end{align*}
$$

Minimization is performed over the variables $x_{ij}^k(q)$, $u_{ij}(q)$, $b_j^k(q)$, and $y_w^k(q)$, where $q \in \{t, ..., t + \Omega - 1\}$, initial backlog $b_j^k(0) = B_j^k$ and initial inventory $y_w^k(0) = Y_w$. The additional variables $u_{ij}(q)$ are introduced to linearize the transportation cost function $\phi(\cdot)$ and represent the number of trucks of capacity $V$ necessary to transport the total amount of products over link $(i, j)$ at time step $q$. To model the step-wise constant transportation cost, these variables have to be restricted to integer values; otherwise a continuous relaxation can be used to model linear transportation costs.

For all time steps prior to $t$, the values of variables are known so that they serve as deterministic data in the above limited look-ahead problem. The same holds for the values of supplies.
Suppose we have supplies $S_k^i(t)$ and demands $D_j^k(t)$ within the considered horizon $\{t, ..., t + \Omega - 1\}$. Consequently, the problem is a mixed-integer quadratic program (in case of stepwise-constant transportation costs) or a quadratic program (in case of linear transportation costs). The model parameters are nicely arranged in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>Number of discrete time slots considered</td>
</tr>
<tr>
<td>$T$</td>
<td>Number of product types</td>
</tr>
<tr>
<td>$S$</td>
<td>Number of suppliers in the network</td>
</tr>
<tr>
<td>$W$</td>
<td>Number of warehouses in the network</td>
</tr>
<tr>
<td>$D$</td>
<td>Number of consumers in the network</td>
</tr>
<tr>
<td>$S_k^i(t)$</td>
<td>Amount of products of type $k$ supplied at time $t$ by supplier $i$</td>
</tr>
<tr>
<td>$D_j^k(t)$</td>
<td>Amount of products of type $k$ demanded at time $t$ by consumer $j$</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>Delay in warehouse $w$ due to consolidation activities</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Horizon length</td>
</tr>
<tr>
<td>$V$</td>
<td>Capacity of a truck</td>
</tr>
<tr>
<td>$c_{ij}$</td>
<td>Costs for sending a truck from node $i$ to node $j$</td>
</tr>
<tr>
<td>$h_w$</td>
<td>Costs per day for storing a product in warehouse $w$</td>
</tr>
<tr>
<td>$\beta_k$</td>
<td>Costs for not delivering product $k$ in time</td>
</tr>
</tbody>
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Table 1: Model parameters

### 2.2 Tactical decision making

In a given collection of suppliers, warehouses, and consumers, let $\mathcal{L}$ be the union of all potential links (directed edges) from suppliers to warehouses, warehouses to consumers, and suppliers to consumers. The tactical problem can then be defined as choosing a subset $\mathcal{U} \subseteq \mathcal{L}$ of a given cardinality $L$ with smallest expected operational cost. In doing so, we tacitly assume that there exists some mapping $\gamma_\mu$ that assigns the expected operational costs $\gamma_\mu(\mathcal{U})$ to the network of chosen links $\mathcal{U}$ when using a particular control policy $\mu$. As it is unclear how this expectation can be computed exactly in the present set-up, we resort to a variant of sample average approximation by Monte Carlo simulation, where we simulate the process for a fixed number of time steps while applying the given policy $\mu$. Here, we use the approximate limited look-ahead policy $\hat{\mu}$ defined above, and refer to the resulting estimator of the expected operational cost as $\hat{\gamma}_{\hat{\mu}}(U)$.

In a bi-objective formulation, this tactical problem is to be solved for any value of $L \in \{L_{\text{min}}, \ldots, |\mathcal{L}|\}$, where $L_{\text{min}}$ is the smallest number of links such that no node is isolated and each warehouse is connected to at least one supplier and one consumer. Our conjecture about this trade-off between the number of chosen links and the operational costs is that the operational costs are only marginally sensitive to a reduction of the number of links from the fully connected network until a critical value, after which the costs increase sharply. In this case, the cost curve would form a pronounced knee. Hence, our particular goal is to determine a representative network from this region, as this would constitute in a sense a best-possible bi-objective approximation. Such a network could be considered “cost-effective” as it would yield close-to-minimal operational costs with a close-to-minimal number of links.

To determine the cost curve, we first develop a bi-objective branch and bound method that computes the minimal cost topology for any given number of links simultaneously. As this method still searches a large part of the solution tree, we additionally propose a heuristic, which is able to generate a satisfactory solution for larger instances.
2.2.1 Bi-objective branch and bound method

We develop a branch and bound algorithm that defines a tree search over a set of binary decision variables \( z_{ij} \) associated with each link \((i, j) \in \mathcal{L}, \) where

\[
    z_{ij} = \begin{cases} 
    1 & \text{if the link from facility } i \text{ to facility } j \text{ is present}, \\
    0 & \text{otherwise}.
    \end{cases}
\]

A pseudo-code description of the algorithm is given below (Algorithm 1). The algorithm maintains and updates a current approximation to the cost curve whose initial values are given in Table 2.

<table>
<thead>
<tr>
<th>( L )</th>
<th>( L_{\min} )</th>
<th>( L_{\min} + 1 )</th>
<th>( \ldots )</th>
<th>( \ldots )</th>
<th>( \mathcal{L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{opt}(L) )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( \mathcal{Z}_{\text{opt}}(L) )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(1,1,1,...,1)</td>
</tr>
</tbody>
</table>

Table 2: Initial state of the table for the minimal costs \( \text{opt}(L) \) and the corresponding solution (network) \( \mathcal{Z}_{\text{opt}}(L) \).

During the search, the \( L \)-entry is updated whenever a network \( z \) with \( L \) links is identified to have lower operational cost than the current best network with \( L \) links (lines 14-16). At each branching operation (lines 19-27), a natural lower bound is given by setting all undecided variables to one as all solutions in this subtree can only be composed of a subset of those links. The subtree can be pruned if this lower bound is not lower than the current best value for any number of links \( L \) for which there are potential solutions in the subtree; otherwise, two children are created by setting the first undecided variable to zero and one, respectively, and added to the list of unexplored nodes \( A \).

2.2.2 Heuristic

Since the branch and bound method evaluates a large number of possible topologies, only small problems can be solved. Hence, we propose a heuristic that evaluates less than \(|\mathcal{L}|\) topologies and still yields a very good solution. The heuristic starts from the fully connected network and approximates the operational costs \( \hat{\gamma}_{\hat{\mu}}(\mathcal{L}) \) when using the control policy \( \hat{\mu} \) over the fixed number of time steps. Then the link(s) that are least used over this time period are deleted. In determining the usage of a link, the value of the product type(s) through this link is taken into account by a weight factor. We define the weighted average daily usage \( P_{ij} \) of the link between facilities \( i \) and \( j \) as

\[
    P_{ij} = \frac{1}{T} \sum_{t=1}^{T} \sum_{k=1}^{K} \frac{\beta_k}{\max_k \beta^k} \cdot x_{ij}^k(t). \tag{6}
\]

In this way, links that provide just-in-time delivery of high value products are not (easily) deleted. This procedure of deleting links is repeated until a minimally connected network remains. A pseudo-code description of the algorithm is given below (Algorithm 2).

3 Computational study

In the previous section, we proposed two algorithms to find a network topology with a close to minimal number of links that is still cost-effective on the operational level. In this section, the performance of both these algorithms is evaluated and compared. Experiments suggest that the heuristic is very effective in constructing a network topology with a small number of links that yields close to minimal operational costs. However, large real-life problems are still intractable.
Algorithm 1 Branch and Bound

1: let $z := (\ast, \ast, \ldots, \ast)$
2: let $LB(z) := -1$
3: initialize the list of unexplored nodes $A$ with $z$
4: while $A$ is not empty do
5:     pick $z$ as the last element from $A$
6:     delete $z$ from $A$
7:     let $p$ be the number of ones in $z$ and $q$ be the number of zeros in $z$
8:     if $p + q = |L|$ then
9:         if $LB(z) = -1$ then
10:             let $c := \hat{\gamma}_\mu(z)$
11:         else
12:             let $c := LB(z)$
13:         end if
14:     if $c < opt(p)$ then
15:         let $opt(p) := c$
16:         let $Z_{opt}(p) := z$
17:     end if
18:     else
19:         if $LB(z) = -1$ then
20:             let $z'$ be a copy of $z$ where all $\ast$ are replaced by 1
21:             let $LB(z) := \hat{\gamma}_\mu(z')$
22:         end if
23:     if $LB(z) < Z_{opt}(L)$ for some $L$ with $p \leq L \leq |L|$ then
24:         let $z''$ be a copy of $z$ where the first $\ast$ is replaced by 0 and append $z''$ to $A$
25:         let $z'''$ be a copy of $z$ where the first $\ast$ is replaced by 1 and append $z'''$ to $A$
26:         let $LB(z'') := -1$ and $LB(z''') := LB(z)$
27:     end if
28: end if
29: end while

Algorithm 2 Heuristic

1: let $\mathcal{U}$ be the fully connected network $\mathcal{L}$
2: while the network is not minimally connected do
3:     apply the policy $\hat{\mu}$ during $K$ time steps and determine $\hat{\gamma}_\mu(\mathcal{U})$
4:     determine usage $P_{ij}$ for each link in $\mathcal{U}$
5:     delete the link(s) from $\mathcal{U}$ with minimal usage
6: end while

Hence, we replace the stepwise cost function by a linear cost function and find similar results much faster. Some large real-life instances are solved for the linear cost structure and results are presented.

To evaluate the performance of the heuristic, a bi-objective relative approximation factor $(\epsilon_c, \epsilon_L)$ is used as a performance measure. The factor denotes the relative deviation from the ideal point, the point composed of the single-objective optima. In this case, those values are known, so we can define the two components as

$$\epsilon_c(\mathcal{U}) := \frac{\hat{\gamma}_\mu(\mathcal{U})}{\hat{\gamma}_\mu(\mathcal{L})} \quad \text{and} \quad \epsilon_L(\mathcal{U}) := \frac{|\mathcal{U}|}{L_{\min}},$$

(7)

denoting the operational costs of the considered topology $\mathcal{U}$ relative to the operational costs in the fully connected network, and the number of links $|\mathcal{U}|$ relative to the number of links in...
a minimally connected network. Even though we are not able to give some a priori performance guarantee of the heuristic, this factor can be used to bound the approximation quality a posteriori.

In addition to evaluating the performance of the heuristic approach, a computational study is performed to investigate the robustness of the generated network topology with respect to changes in the supply and demand distributions. Results suggest that a “heuristic network” with a satisfactory \((\epsilon_c(\mathcal{U}), \epsilon_L(\mathcal{U}))\) value is cost-effective as long as the means of supply and demand distributions remain the same. Finally, we formulate the hypothesis that increasing the initial inventory enables to compensate for the backlog costs embedded in a particular supply and demand time series. The hypothesis is confirmed by experimental results.

For the logistics networks in this paper, we consider \(S\) suppliers, \(W\) warehouses, \(D\) consumers and the distribution of \(T\) product types. The networks considered typically consist of few suppliers and warehouses and a lot of consumers, such that \(W < S \ll D\). Furthermore, we generate stochastic time series of \(T \cdot S\) supplies and \(T \cdot D\) demands from uniform distributions with random means and variances for 100 time steps. We assume that supplies and demands are only known 2 days in advance, so the window size \(\Omega\) is equal to 3. Moreover, we focus on networks of relatively small geographical extent. Therefore, it is assumed that products can be transported from any arbitrary node to another within one day. In addition, we assume that the delay for going through warehouse \(n\) is equal to one day, which implies that \(\tau_w = 1\). The costs are chosen such that the proportion of transportation and storage costs \(c_{ij} : h_n = 1 : 0.6\) on average and the early/late delivery costs \(\beta^k\) are randomly chosen between the bounds of 0.001 and 100.

### 3.1 Comparison: Branch and bound versus heuristic

First, we compare the results of the branch and bound method and the results of the heuristic applied to the stepwise cost model. Hence, we generate different problem instances (supply and demand distributions as well as spatial location of network nodes) for many small network configurations and run the branch and bound method as well as the heuristic. Comparison of the resulting cost curves of operational cost versus number of links show that the heuristic generates a topology that is close to optimal.

To illustrate this we use a network with \(S = 2\), \(W = 1\), \(D = 3\) and \(T = 1\). A hundred sets of supply and demand time series and graphical supply, demand and warehouse locations are generated. Next, for each of these sets, a network topology is constructed with both the branch and bound, and the heuristic method.

Results for both the branch and bound method and the heuristic are shown in Figure 2. For clarity of presentation we only depict results for ten different parameter sets, which give a proper representation of all hundred performed experiments. As can be seen from this figure, both methods generate equal operational costs for the fully connected network (number of deleted links equals zero). From Figure 2, it also appears that the critical value in the efficient frontier found by the heuristic occurs at 5 or 6 deleted links, whereas it always occurs at 6 deleted links for the networks generated by the branch and bound approach. The difference in the number of links in the resulting network for these instances is thus at most 1 link. This suggests that the heuristic finds a satisfactory solution for small network instances. Since the CPU time of the branch and bound grows extremely fast with increasing network size, we have to rely on the heuristic for larger instances.

The heuristic, applied to the stepwise cost model, enables to find a close to optimal topology for a network with up to 4 suppliers, 3 warehouses, 25 consumers and 6 product types (\(S = 4\), \(W = 3\), \(C = 25\) and \(T = 6\), respectively). Due to the non-linearity, larger real-life problems are still intractable. We therefore remove the integer variables \(u_{ij}\) from the model and introduce linear transportation costs \((\phi(x) \equiv Vx)\). We apply the heuristic to this linear model, which is now able to construct cost-effective topologies for large real-life network sizes. In the next
section, results of the heuristic for the stepwise and linear model are compared. Additionally, we present results of the heuristic for the linear model for large real-life network sizes.

3.2 Performance of the heuristic for large networks

Results for both the linear and the stepwise model are presented in Figure 3 for two different network configurations. For larger network sizes, we have to rely on the heuristic model with linear costs. Two cost curves for large real-life networks are presented in Figure 4. Costs are plotted on a logarithmic scale versus the number of deleted links. In these plots, zero deleted links corresponds to the fully connected network, whereas the maximal number of deleted links corresponds to a minimally connected network.

All these curves (in Figures 3 and 4) exhibit the same typical characteristics from the left to the right: at first, a large number of links can be deleted without affecting the costs significantly: Up to 70% of the links is not used at all in the first heuristic iteration and is thus deleted at once. Still, about 15% of all the links can subsequently be deleted without substantially affecting costs. Then, after about 85% of the total number of links that can be deleted, costs start increasing slightly with the number of deleted links. Finally, when about 90-95% of the total number of links that can be deleted are actually deleted, costs start increasing dramatically. Below the figures the $\epsilon_L$-value are given for an upper bound on the $\epsilon_c$-value of 1.01, which means that the heuristic is stopped if deleting one more link would lead to operational costs that are more than 1% higher than the operational costs in the fully connected network.

The comparison between the results of the linear and stepwise model show that in the fully connected network, as expected, the costs for the stepwise model are slightly higher than the costs for the linear model (due to the logarithmic scale this can hardly be noticed in Figure 3). Furthermore, Figure 3 shows that for both models the dramatic cost increase starts at approximately the same number of links. Further analysis (not depicted in the figures here) shows that for the performed experiments, the set of deleted links at the critical value is approximately the same for both models. A large number of experiments, not presented in this paper, exhibit the same characteristics, suggesting that the heuristic for the linear model is a quite good approximation of the heuristic with the stepwise model.

Apparantly, a topology with only a small number of links suffices to gain close to minimal
operational costs. Since only a small number of links remain we can also conclude that the resulting topology is also close to optimal. In (Jordan & Graves 1995), similar characteristics are presented for a manufacturing system with deterministic supplies and demands. The results in (Jordan & Graves 1995) suggest that a small flexibility (each plant produces only a couple of product types) can almost achieve the benefits of total flexibility (each plant produces all product types). Furthermore, it should be noticed that the curves in Figures 3 and 4 exhibit approximately horizontal plateaus. The reason for this is that not one, but a group of links has to be deleted to significantly affect the costs.

3.3 Statistical robustness analysis

We now formulate the hypothesis that networks generated for a particular supply and demand time series, are insensitive to changes of second order moments of supply and demand distributions. Thus, if information about the individual means of supplies and demands is available a priori, we only need to generate a stochastic time series with these means to find a cost-effective
topology for other scenarios with these means. An additional experiment suggests that these stochastic time series, rather than deterministic ones, are required to generate a robust topology. Furthermore, we perform an experiment suggesting that the heuristics applied to the linear model finds a topology, which is efficient with respect to the economy of scales principle (consolidation of product flows to induce full truckload shippings). Finally, as we optimize over a finite period of time $T$, we have to address the influence of the initial inventory. We demonstrate that the steady state value of the costs in the fully connected network converges to the minimal transportation plus storage costs, as initial inventory increases.

### 3.3.1 Sensitivity

To allow for an automatic statistical sensitivity analysis we define a representative “heuristic network” to be the smallest network $U$ determined by the heuristic, whose operational costs are within a factor $F$ of the operational costs of the fully connected network, i.e., where $\epsilon_c(U) \leq F$. Deleting one more link will increase the costs above that level. We then perform the following statistical analysis: We choose fixed mean supplies and demands for a network with $S = 4$, $W = 3$, $D = 20$ and $T = 5$ and generate stochastic supply and demand samples (for 100 time steps). Based on this, we determine the “heuristic network” and fix its topology, i.e., its link structure $U$. For this network $U$, we then compute the operational costs for supply and demand time series from twenty different distributions (with different second order moments but same means), record the resulting approximation factor $\epsilon_c(U)$ in each case. Finally, the mean values $\bar{\epsilon}_c$ and $\bar{\epsilon}_L$ of the twenty instances are determined.

The above described experiment is repeated for 99 other heuristic networks, each generated from particular means of supply and demand. Figure 5 shows four scatter plots (each for a particular value of $F$) of the hundred mean values $\bar{\epsilon}_c$ versus $\bar{\epsilon}_L$. As expected, the robustness decreases as $F$ increases (since then the number of links in the heuristic network decreases which makes the network more sensitive). We define the network to be robust if the mean value of $\epsilon_c$ (over 20 experiments) is at most 1.2 with a frequency of 95%. The scatter plot in Figure 5a shows that $\epsilon_c$ is strongly clustered near one indicating that, with high likelihood, our process of choosing a heuristic network (with $F = 1.001$) will find a network that is a “very good” network even when the supply and demand time series vary randomly with respect to second order moments. The histogram for the case where $F = 1.01$ (Figure 5b) is still satisfactory since more than 95% of the considered cases results in an $\epsilon_c$ value equal to or smaller than 1.2. The results for $F = 1.05$ (Figure 5c) and $F = 1.1$ (Figure 5c) do not satisfy our definition of a robust network anymore. For these values of $F$, several means and standard deviations of $\epsilon_c$ are even outside the figure range (larger than 3). We choose as “heuristic network” the one with $F = 1.01$, since (i) the accompanying results fulfill our definition of a robust network, and (ii) the generated heuristic networks have fewer links than the case where $F = 1.001$. These results suggest that the “heuristic network” ($F = 1.01$) indeed is robust to changes in second order moments of supply and demand distributions.

The corresponding values of $\epsilon_L$ suggest that the generated heuristic networks on average consist of 2.5 times the number of links in a minimally connected network. This means that each network node is on average connected to two or three other network facilities. Apparently, this limited amount of connectivity suffices to provide a large portion of the operational efficiency and robustness of the fully connected network. Another interesting result is that the number of links in the “heuristic network” depends on the product mix. A circle in Figure 5 represents an instance with a product mix with only one high value product and in this case four low value products, whereas a plus represents an instance with a product mix with at least two high value products. The results suggest that more links are required when more high value products are present. We give the following explanation for this phenomenon: the structure of the “heuristic network” is mostly determined by the number and locations of suppliers and consumers of high value products. When more high value products are present, in general the number of suppliers
To check the sensitivity of the “heuristic network” to the first moments of supply and demand and consumers of high value products increases, and thus more links have to be established to provide just-in-time delivery.

Figure 5: Sensitivity “heuristic network” to 2nd order moments. ‘o’ product mix with 1 high value and 4 low value products, ‘+’ product mix with more than 1 high value product.

Figure 6: Empirical cumulative distributions showing the sensitivity of the heuristic networks to changes in i) variances and ii) means of supplies and demands.
distributions, we perform the following experiment: we take the hundred heuristic networks, where $F=1.01$, and hundred accompanying results of $\epsilon_c$ from the previous experiments where the means of supplies and demands stay the same, but the variances change (Figure 5b). In addition, for each of the 100 heuristics networks we generate supply and demand time series from completely different distributions and determine the corresponding $\epsilon_c$. Figure 6 depicts two empirical cumulative distribution functions of hundred $\epsilon_c$ values generated from i) time series with different variances, but the same means as the “heuristic network” originally was generated from (solid line), and ii) time series with different means (dotted line). The solid line again confirms that the “heuristic network” is robust to the variances since the cumulative distribution grows to 100% very fast starting at $\epsilon_c = 1$. The cumulative distribution function for the instances with different means grows much slower. Furthermore, only 80% of the $\epsilon_c$ values are smaller than or equal to 10. These results suggest that indeed the “heuristic network” ($F = 1.01$) is sensitive to changes in the means of supply and demand distributions.

3.3.2 Deterministic versus stochastic approach

As the results from the previous subsection suggest, only the supply and demand averages are required to construct a robust topology. This however does not imply that solving the deterministic problem based on these averages, results in a robust topology. On the contrary, (random) stochastic supplies and demands are required to create back-up links that are not generated in the deterministic approach. This is illustrated by the following experiment. We consider 3 suppliers, 2 warehouses and 15 customers in a distribution network with 5 product types, i.e. $S = 3$, $W = 2$, $D = 15$, and $K = 5$. We first randomly generate $3 \cdot 5$ supply and $15 \cdot 5$ demand averages. Now we construct network topologies in the following two ways: the heuristic applied to i) the deterministic case based on these averages, and ii) stochastic supplies and demands (with these same averages and additionally randomly generated second order moments). In both cases $\epsilon_L = 1.01$ for arriving at a topology. Next, 31 new supply and demand time series (of 100 time steps each) with the same averages but different second order moments are generated. The performance of both found topologies for the new time series (relative to the performance of the fully connected network for the new time series) can now be compared by means of the value of $\epsilon_c$. Figure 7a depicts the results of 31 topologies ($F = 1.01$) each subject to 31 time series. Results are expressed by values ($\epsilon_L$, $\epsilon_c$). The (extreme) values of $\epsilon_c$ for the deterministic approach (circles) are much larger that the ones for the stochastic approach (stars). Apparently, the approach based on stochastic supplies and demands leads to much more robust network topologies than the approach based on deterministic supplies and demands.

3.3.3 Performance of the heuristic using linear transportation costs

Since the heuristic using the stepwise model can only solve medium-size instances, we have to rely on the heuristic using linear transportation costs (a cost model without an economy of scale principle). However, we have implemented a kind of a consolidation aspect in our heuristic instead: the utilization of a link is weighed according to the type(s) of product flowing through it. The higher the value of product $k$, the higher the weigh factor $\beta_k$. In this way, we prevent links that transport high-valued product(s) from being deleted and induce links with low-valued products to be deleted. The low-valued products will eventually be added to the links with high-value products. Although we realize that this consolidation effect is different from the one in the stepwise model, and as a consequence the daily shipping decisions may be different for both approaches, the constructed network topologies turn out to be very similar. Since this is exactly what we are looking for, namely a good network topology, the heuristic with the linear model can be used for solving large instances, e.g. $S = 8$, $W = 4$, $D = 75$, having $2^{362} \approx 3.63 \cdot 10^{280}$ possible topologies.
The following experiment is developed and performed: We again consider 3 suppliers, 2 warehouses and 15 customers in a distribution network with 5 product types, i.e. $S = 3$, $W = 2$, $D = 15$, and $K = 5$, and use the found 31 heuristic topologies from the first experiment in this report. Then, for each of these topologies, we run the stepwise model for i) the found topology and ii) the fully connected network for thirty-one supply and demand time series (the same as in the first experiment) with the original averages, but different second order moments, and compare both costs. Corresponding values of $(\epsilon_L, \epsilon_c)$ are shown in Figure 7b. We see that the values of $\epsilon_c$ are still satisfactorily small (major part below 1.2) and closely resemble the values for $\epsilon_c$ (stars) in Figure 7b. Apparently, running the heuristic using the linear cost model results in a topology, which can efficiently be used from with an economy of scales point of view.

![Figure 7: Validation of the heuristics performance.](image)

(a) Deterministic versus stochastic approach, $S = 3$, $W = 2$, $D = 15$, $K = 5$, $F = 1.01$.  
(b) Performance of the stepwise model for the topology found with the linear model, $S = 3$, $W = 2$, $D = 15$, $K = 5$, $F = 1.01$.

### 3.3.4 Inventory flexibility

In all previously performed experiments the parameter of initial inventory $y_k^w(0)$ has been set to zero. It turns out that this initial inventory position is the only controllable parameter that can influence the operational costs for running the fully connected network. Consider the black dotted lines in Figure 8a and 8b, which show results of the heuristic for $S = 4$, $W = 3$, $D = 20$ and $T = 5$, for respectively two different supply and demand time series with equal first moments and $y_k^w(0) = 0$. The fact that the means of supply and demand are equal implies that approximately the same amount of products is shipped along time period $T$, and thus approximately the same transportation costs are present for both these cases. Nevertheless, a large difference in costs for the fully connected network can be noticed. This difference can therefore only be caused by backlog costs, which are induced by the specific time series. For instance, a time series, which exhibits a peak in demand at the beginning of period $T$, always leads to backlog costs, even in a fully connected network. Therefore, we pose the hypothesis that the only controllable parameter that influences the costs for the fully connected network is the initial inventory. Increasing the initial inventory will compensate for early backlog, which eventually will lead to convergence to the inevitable transportation costs.

To confirm this hypothesis, we perform the following additional experiments for the two supply and demand time series as mentioned above: we generate a supply and demand time series, set the inventory position in each of the warehouses to zero and determine the operational costs dependent on the number of deleted links (black dotted lines in Figure 8a and 8b). Next, we perform two additional optimizations for the same supply and demand time series where...
1.0% and 2.5% of the total shipped amount of products is present in the warehouses as an initial condition, respectively. The results of these experiments are depicted in Figure 8a and 8b. As one can see, for both cases, the total costs of the fully connected network indeed converge to the transportation and storage costs as initial inventory increases.

4 Conclusions and recommendations

In this paper we considered transportation networks from the point of view of a fourth party logistics provider, who deals with the problem of distributing different types of products from suppliers to consumers via transportation links. Warehouses between the suppliers and consumers may be used to compensate for the stochastic behavior of supplies and demands and to consolidate different products. The amounts of these supplies and demands are assumed to be uncontrollable for the logistics provider. With only information about the supplies and demands a few days in advance, the logistics provider has to decide on which transportation links to use for a long period of time (tactical level) and how much of which products to ship through them each day (operational level).

We formulated a bi-level joint network design and network operation problem: at the upper (tactical) level, the network topology has to be constructed, and at the lower (operational) level routings and schedules for daily shipments have to be decided on. A model predictive control with a rolling horizon (MPC) was used as decision model on this operational level. A heuristic was proposed to construct the topology dependent on the operational decisions and compared to a bi-objective branch and bound method. The consolidation of product types so as to leverage on the economy of scales principle is taken into consideration in this network design heuristic.

Experimental results reveal that the cost of such a near-optimally operated network as a function of the number of links in the network stays almost constant as the vast majority of network links is removed and explodes once the link number has decreased below a critical value. The experiments show that our heuristic determines a network that has close to optimal costs with a very low number of links (typically about 10% of all links that can be deleted before a minimally connected network remains).

Furthermore, experimental results suggested that the resulting topology is insensitive to second order moments of the individual supply and demand distributions. Hence, information about the means of supplies and demands over a certain time period suffices to generate a close to optimal network topology robust to any supply and demand scenario with the same means.
References


