El Farol Bar Problem

Externalities
Externality = “any situation in which the welfare of an individual is affected by the actions of other individuals, without a mutually agreed-upon compensation” (Easley & Kleinberg, 509).

- Negative externality: welfare decreases
- Positive externality: welfare increases

Why El Farol has negative externalities: as the number of people going increases up to 60, welfare increases, but after 60, welfare decreases (less pleasant when it’s crowded).

Situation with positive externalities: a company with 100 people wants to use a company-specific social networking site. 60 is the minimum number necessary for it to be worth using. As more people join the site, it becomes more valuable, maxing out at 100 users (Easley & Kleinberg, 534).

Comparison: as values increase up to 60, welfare in each situation will increase. However, for the El Farol problem, welfare decreases at any value after 60, while for the social networking site, welfare increases after 60. So, because one has positive externalities and one has negative, welfare behaves differently after 60 for each.

Nash Equilibria
In this case, the game is only played once and 100 people are interested in attending. With this in mind, each person has two possible strategies, to Go or to Stay. The payoffs of these strategies are:

\[ \text{Stay} = 0; \quad \text{Go} = \begin{cases} \text{x} > 0, \text{if at most 60 people go} \\ \text{-y} < 0, \text{if more than 60 people go} \end{cases} \]

There are many pure strategy equilibria, but only when exactly 60 people Go and exactly 40 people Stay. Since all 100 people are interested in going and are considered identical, it is more useful to construct identical mixed strategies for each player.

When trying to find the Nash Equilibria, there must be no incentive for any player to deviate from randomizing their choice based on a probability. Therefore, if every person has the same probability of picking Go, and the payoff of Stay is the same as the payoff of Go, there is no incentive to change strategies. (Easley & Kleinberg, 536)

Let \( p \) represent the probability of choosing Go. Therefore, how \( p \) is chosen depends on the value of the payoffs \( x \) and \( y \), which can be altered. Such alterations may arise from the severity of the discomfort a crowd of over 60 creates and the magnitude of the utility the music creates. So, choose \( p \) such that the expected payoff of Go is equal to 0. This may be represented as the following equation, then simplified with a substitution: (Easley & Kleinberg, 537)

\[ x \cdot \Pr(\text{at most 60 go}) - y \cdot \Pr(\text{more than 60 go}) = 0 \]

\[ \Pr(\text{more than 60 Go}) = 1 - \Pr(\text{at most 60 Go}) \]

\[ \Pr(\text{at most 60 go}) = \frac{y}{y + x} \]

Two Players
Each player wants to attend as long as the other player doesn’t. Pure-strategy equilibria is shown when the decisions are opposite while for mixed equilibria, they make their decision as the following probability holds true: \( p = \frac{x}{x + y} \)

El Farol Bar Problem vs. Hawk-Dove Game
This problem correlates to the hawk-dove game because in the El Farol Bar problem, two players are trying to make their actions different. The similarities can be seen in the following payoff matrices:

El Farol Bar Problem vs. Coordination Game
In this game, both players would be trying to make their actions similar. This is not the case with the El Farol Bar game because the two players are trying to make the opposite decision in order to have a higher payoff instead of coordinating to make the same. The coordination game can also have unequal payoffs assuming that one strategy will have a higher payoff than the other. (Easley & Kleinberg, 537-38)

Repeated El Farol Problem
In a repeated El Farol game, the players must decide, based on the previous bar attendance records, whether they will go to the bar on the upcoming Thursday. All players have the same information, and from this, each player comes up with a forecasting rule (a function that maps the past history of a play into a prediction about the future actions of all other players). The player will make a decision that optimizes his payoff using his forecasting rule. The most common way to analyze the repeated El Farol problem is to create a function that predicts the next audience size based on all of the previous audience sizes. However, if all players use the same forecasting rule, then everyone will make a bad prediction. Thus, this problem becomes more complicated, as there must be a variety of forecasting rules used by individual players. The analysis of this is complex, and has been carried out mathematically and with a computer simulation. Generally, the bar attendance converges to a state where the attendance varies around 60 patrons. (Easley & Kleinberg 538-540)