Introduction

The identification of fingerprints depends on three levels of detail. Level 1 detail is the overall pattern of the fingerprint [1]. Fingerprints may have a loop pattern where the ridges enter and exit on the same side, an arch pattern where the ridge enters on one side and exits on the other, and a whorl pattern where the ridges form a more circular pattern [1]. These patterns are rooted in embryonic development and the positioning of an embryonic structure known as a volar pad [1]. The size, shape, and point of regression of the volar pad play a role in determining the ridge count, and overall pattern of the fingerprint [1]. During embryonic development, it is this the primary and secondary ridge endings of the epidermis that help give rise to the patterns of a fingerprint [1]. Level 2 detail is the minutiae in the fingerprint, the bifurcations, ridge endings, and islands [1]. The minutiae arises due to the manner in which the fingerprint forms. The pattern begins growing around the nail bed, below the delta, and at the core of the fingerprint [1]. Minutiae arise when these three segments come together and create one larger pattern. Level 3 detail is the pores and sweat glands that may be seen in a high quality fingerprint [1]. These details arise due to the glands that travel up through the cell layers from the body [1]. Altogether, the formation of the level of detail found in fingerprints is deeply rooted in embryonic and fetal development [1]. Understanding the method by which the fingerprints are formed in nature provides insight on how fingerprints could be generated synthetically through mathematical modeling.

Fingerprint Statistical Analysis: False Matching and Synthetic Fingerprint Creation

All fingerprints are unique, and the uniqueness of each fingerprint is created by spatially distributed patterns, ridgelines, and minutiae. In a study done by Stephen Taylor, Emma Dutton, Patrick Aldrich, and Bryan Dutton, the
Geographic Information System (GIS) is used to analyze fingerprint features, and fingerprint matching techniques. The ultimate goal of this study is to use GIS software to estimate false match probabilities, as well as quantify the distribution of minutiae and ridge lines in a coordinate spaced fingerprint.

The Geographic Information System used in this research is composed of hardware and software components, which capture data through grids and vectors on a three dimensional coordinate system [2]. The Geographic Information System was developed for computerized land management, and eventually was used for surgical procedure characterization, as well as crime pattern analysis. In terms of fingerprint research, GIS is used to look at the spatial patterns to estimate false-match probabilities, and create a framework for characterizing and quantifying fingerprint characteristics [2]. For sample prints to be analyze by the GIS technology, they are rated on quality, processed with photoshop enhancements, and oriented with the interphalangeal crease as the horizontal determiner [2]. The Universal Latent Workstation, a software application, is used to identify bifurcations and ridge endings as minutiae [2]. Latent print examiners then ensure that all the collected data is accurate [2]. Using scripts written in Python, the given fingerprint is labeled as one of the three patterns, a loop, whorl, or arch [2]. Following the pattern labeling, the delta is marked, where three ridges converge with each other [2].

It was found that an average of 85 minutiae were present in a single fingerprint, when comparing all pattern types, with a greater density of minutiae in the lower region [2]. Each pattern type differed between minutiae content, with tented arches having the least, an average of 65.2, and double loop whorls have the most, an average of 99.5 minutiae [2]. Bifurcations were more common close to the core, while ridge endings were more common close to the delta region [2]. In terms of minutiae type, ridge endings outnumbered bifurcations, by a factor of 1.4 [2]. These minutiae mainly occur below the core, and are seen more in more complex patterns [2]. It is determined that minutiae distribution are not necessarily random, and are a key factor in accurately creating a synthetic fingerprint.

In terms of false matching, the Monte Carlo method is utilized to simulate the ability of finding locations of minutiae that are similar [2]. There is a negatively correlated relationship between the number of minutiae and the probability of matching a fingerprint, such that when the number of minutiae to match is increased, the probability of finding a matching print significantly decreases [2]. This exemplifies the idea that the possibility of a false match is extremely low when there are many minutiae selected to be compared [2]. To find matching minutiae in a fingerprint, the print is divided into cells, and the minutiae in each cell is distinguished from one another [2]. As the cells are compared, the location of the minutiae present is compared by location on an X-Y coordinate system, and when the minutiae match, it is vital to determine whether or not the match minutiae are of the same type [2]. In terms of synthetically creating fingerprints through computer applications, it is important to ensure that each fingerprint created varies with minutiae placement and type.
Overall, fingerprints are defined by various characteristics, mainly their pattern formation and minutiae placement. The two are related, such that the number of minutiae present is due to the type of pattern. With the use of Geographic Information Systems, along with supporting applications, the density and placement of minutiae were determined, which are characteristics needed to be capable of synthetically creating fingerprints. Since minutiae are not uniformly distributed, nor random, there are still underlying biological processes to ultimately determine the spatial distribution and pattern of minutiae.

**SFinGe: Synthetic Fingerprint Generation**

Cappelli discusses a novel method of Synthetic Fingerprint Generation (SFinGe). In this method, a master fingerprint is generated from an actual fingerprint using the fingerprint shape, a directional map, a density map, and a ridge pattern formation [3]. The master fingerprint is then altered according to a fingerprint impression. This is done through selection of the region of contact, variation in the average ridge thickness, distortion of the fingerprint, processing noise and rendering, global translation or rotation, and finally, generation of a more realistic background [3]. This method of generating synthetic fingerprints is a novel, zero cost approach to generating large databases of fingerprints, but there are several limitations to this method. First, the ridge thickness is constant throughout the entire synthetic image whereas in natural acquired fingerprints, the ridge thickness varies throughout the image [3]. Additionally, noise in the fingerprint is distributed uniformly across the image, but in actual prints, the noise may be clustered in particular areas [3]. Finally, the Level 3 details of the pores are not very accurate because these details are based on random generation in SFinGe, whereas in nature, these features have a distinct pattern amongst impressions from the same finger [3]. Altogether this approach to the synthetic generation of fingerprints is an interesting concept, but the practicality of this process is limited due to the need for acquired fingerprints to generate the master fingerprint. Also, this method of fingerprint generation is novel, but the generated fingerprints will generally be modifications of the master fingerprint and share similar characteristics in terms of general pattern and minutiae with the master fingerprint.

Using the master fingerprint idea, in 2007, Cappelli et al. expanded upon this idea by designing an experiment to test the concept. Using the forefinger and middle finger on both hands from 30 volunteers, it was possible to obtain 120 different fingerprints [4]. The data from the volunteers was collected in three distinct groups, one was naturally obtained, and the other two had specific perturbations such as exaggerated fingerprint displacement and rotation and moistened or dried fingers [4]. From these fingerprints, the global pattern, minutiae distribution and type, and ridge density were extracted and four synthetic fingerprint images were created using different frequency values for the ridge patterns [4]. The synthetic prints bore some resemblance to the master, ISO,
fingerprint, but there are some differences like the pore distribution and minutiae formations that created new fingerprints [4].

The four synthetic prints were then compared to the ISO print to determine if the synthetic prints could be used as a masquerade attack against current fingerprint matching systems [4]. Nine state-of-the-art fingerprint matching algorithms were tested at three threshold levels against four different hypotheses [4]. The three thresholds ($\tau$) chosen were a False Match Rate (FMR) of 1%, 0.1%, and 0% [4]. The four hypotheses were BASE, MinType, SingPos, and OrImg [4]. In BASE, only the required fingerprint information from the standards were used without minutiae type information. MinType included the mandatory information from the standards used for fingerprint identification and minutia type information was made available [4]. SingPos was the same as BASE, but information about the core and delta was included in the extended data [4]. OrImg was also the same as the base, but the original information about the orientation of the image was included in the extended data [4]. The attack was considered a success if at least one of the comparisons of the synthetic image against the ISO print lead to a score greater than $\tau$ [4].

The results from this experiment are relatively surprising. Among all security levels and hypotheses, the lowest average percentage of 79.35% successful attacks occurred at FMR = 0% under the MinType hypothesis [4]. The highest percentage of 99.63% successful attacks occurred at FMR = 1% under the OrImg hypothesis [4]. Under the BASE hypothesis, at the middle security level, FMR = 0.01%, the average percent of successful attacks was 90.56% [4]. Under the MinType hypothesis, the percentage of successful attacks at the same security level is 89.54% [4]. This implies that having knowledge of minutiae does not greatly impact the accuracy of the algorithms [4]. With the knowledge of the singularity positions (SingPos) and fingerprint orientation (OrImg), the probability of a successful attack increases slightly with knowledge of singularity and increases significantly with knowledge of the fingerprint orientation [4].

Two major questions were brought up from the ability to generate synthetic fingerprints [4]. Is it possible to fool a trained expert using a synthetically created fingerprint? Is it possible to masquerade an attack on a fingerprint recognition software? The synthetic fingerprints have slight distinctions that, to a trained expert, differentiate them from actual fingerprints. These distinctions include the local shape of the minutiae and the distribution of the pores along the ridges [4]. Secondly, the use of these synthetically generated fingerprints has an average of 81% success against a high security level system (FMR = 0%) and 90% successful at a medium security level (FMR = 0.01%) [4]. From this research, it is possible to determine that more robust algorithms must be created in order to prevent an attack using computer generated prints [4]. In addition, the methodology used to generate the synthetic fingerprints could be altered to make these prints more realistic using the ridge counts from the print [4]. To further improve the prints, the process of generating the fingerprints could be altered in order to prevent the creation of synthetic additional minutiae [4]. While these synthetic fingerprints are highly similar to the master fingerprints from which

4
they were derived, it could be possible to use information gathered from these prints in order to develop methods to generate more unique fingerprints.

**Turing Patterns**

Reaction-diffusion equations may be used to describe patterns that form in nature such as lines in sand and fingerprints. Kücken uses a reaction diffusion convection system while others may use a glycolysis reaction model to create models that represent fingerprints [5]. A reaction diffusion system is defined by equations for two species as seen in (1) [5].

\[
\begin{align*}
\frac{du_1}{dt} - \nabla^2 u_1 &= \gamma \cdot f(u_1, u_2) \\
\frac{du_2}{dt} - d\nabla^2 u_2 &= \gamma \cdot g(u_1, u_2)
\end{align*}
\] (1)

In the reaction-diffusion system, \( u_1 \) and \( u_2 \) are the concentrations of the chemical species present in terms of the functions \( f \) and \( g \), \( d \) is a dimensionless diffusion coefficient, and \( \gamma \) is a dimensionless constant for the system [6]. A reaction-diffusion system subject to the conditions listed in (2), generates a Turing instability. [5]. This theory of pattern development has assisted in the explanation of complex biological patterns such as spotted animals and morphogenesis issues [5].

\[
\begin{align*}
fu_1g_{u_2} - fu_2g_{u_1} &> 0 \\
fu_1 + gu_2 &< 0 \\
dfu_1 + gu_2 &> 0 \\
(dfu_1 + gu_2)^2 &> 4d(fu_1gu_2 - fu_2gu_1)
\end{align*}
\] (2)

For these conditions, \( fu_1 \) and \( gu_1 \), are derivatives of the reactions, \( f \) and \( g \), with respect to the concentration variable \( u_1 \), and \( fu_2 \) and \( gu_2 \) are derivatives of \( f \) and \( g \) with respect to the concentration variable, \( u_2 \) [5, 7]. These restrictions were considered at an equilibrium point defined as \( f(u_1, u_2) = g(u_1, u_2) = 0 \) [5].

The glycolysis equations for predicting pattern formation are given in (3), where \( \delta \) and \( \kappa \) are dimensionless parameters of the model [5, 8].

\[
\begin{align*}
f(u_1, u_2) &= \delta - \kappa u_1 - u_1^2 \\
g(u_1, u_2) &= \kappa u_1 + u_1^2 - u_2
\end{align*}
\] (3)

The steady state equilibrium points of this system are given by \((u_1, u_2)_0 = \left( \frac{\delta}{\kappa + \delta}, \delta \right) \) [5]. The use of these constraints and the glycolysis equations establish the Turing space [5, 8]. In order to better model the strain of the fingertip surface as seen in embryonic growth, it is possible to modify the morphogens
present in the domain in terms of the surface, $S$, being strained according to the normal, $N$, and the molecular concentration of $u_2$ at each point giving rise to Equation 4, where $K$ is a constant concerning the growth rate [5].

$$\frac{dS}{dt} = Ku_2(x, y, z) N \tag{4}$$

Incorporating the strain from Equation 4 into the reaction-diffusion system from Equation 1 gives rise to a new system that accounts for the convection and dilatation of the domain, shown in Equation 5, where $\nabla \cdot (u, v)$ includes the convection and dilatation from the growth of the domain given by the velocity, $v = \frac{ds}{dt}$.

$$\frac{du_1}{dt} + \nabla \cdot (u_1v) - \nabla^2 u_1 = \gamma \cdot f(u_1, u_2) \tag{5}$$

$$\frac{du_2}{dt} + \nabla \cdot (u_2v) - d\nabla^2 u_2 = \gamma \cdot g(u_1, u_2)$$

It is possible to solve the reaction-diffusion convection system using the finite difference method, using a Newton-Raphson method to solve the nonlinear problem [9, 10]. In terms of modeling fingerprints, when the chemical factors, the morphogens, are taken to be an activator and inhibitor, it is possible to generate Turing patterns [5]. In particular, these solutions may be used to model both two dimensional fingerprint impressions and three dimensional fingers. The solution to the system under different constraints may be used to model the pattern formation on a three-dimensional mesh [5]. By taking images from different time points in the system, it is possible to see the evolution of the patterns formed under different constraints of the system [5]. The ability to visualize the process of pattern formation could eventually give insight into the biological processes under which fingerprints are formed. If this is possible, the insight into pattern formation could give further information concerning the generation of realistic artificial fingerprints. In addition, the ability to use this system in two dimensions could lead to the direct creation of two dimensional images that resemble fingerprints. In order to improve this reaction-diffusion model, it may be necessary to change the number of morphogens and experiment with the parameters to optimize the generated solutions. Further research into these types of systems could give rise to the ability to create a database of synthetic fingerprints.

**Elasticity**

Understanding the physics of fingerprints is essential to knowing how the patterns of fingerprints develop. What determines the ridge system and minutiae is a complex system of various pressures on an elastic material, in this case, skin. It is important to know how pressures on a system can affect and deform the materials. Consider a system where the distance between two points is described by $dx_i$ and the displacement during deformation as $du_i$. Then the
distance between the two points after deformation is \( dx' \). After some equation manipulation, we find a property known as the strain tensor, as seen in Equation 6, for a two-dimensional system [11].

\[
u_{ik} = \frac{1}{2} \left( \frac{d u_i}{d x_k} + \frac{d u_k}{d x_i} \right)
\]

(6)

The strain tensor helps to describe the strain in a system. Strain is found in any system where the length of an object or material has been deformed [11]. Exactly how this strain affects said object is dependent on the ability of the object to withstand deformations. If the force on an object is greater than what that object can withstand, there is deformation [12]. If the deformation is permanent, it is called plastic deformation; if not, then it is elastic deformation [12]. There are three main types of stress, but the stress most applicable to fingerprints is compressive stress [12]. Compressive stress is when force is pushing inward on a material and in the case of fingerprints, this is largely caused by a high growth rate in the skin and not surface area on the finger for all the skin to fit without deforming [1]. Using thermodynamics, Hooke’s Law, and Poisson’s ratio, it is possible to describe exactly what forces are being exerted in a system with compressive stress [12]. In his 2004 paper, Kücken describes fingerprints as a process where compressive stress causes the skin to buckle in the basal layer, thus forming ridges [13]. The basal layer is a thin layer between the dermis and epidermis. Basal layer skin cells reproduce exceptionally fast causing a compressive force in the layer [13]. When these compressive forces on the basal layer exceed the forces exerted on it by the dermis and epidermis, buckling occurs, causing ridgelines to form in the skin [13].

Agents

Because of the modern use of DNA for many forensic and genetic applications, research on fingerprints has slowed significantly in the last few decades. Because of this, fingerprint models from even 100 years ago are important in today’s research. The past several years have yielded several papers by Michael Kücken and other researchers investigating a model for the formation of friction ridge skin [13, 14, 15]. As outlined previously in the biological summary, epidermal ridge patterns begin in the 10th week of fetal development [1]. Primary ridges form in the basal layer, providing the foundation for the rest of the ridge patterns on the surface of the skin [1]. The ridges are caused by two primary forces: the large growth rate of the epidermis and the change in finger shape due to the shrinking of the volar pads [1].

There have been many possible explanations for what causes the ridge pattern and Kücken suggests an agent based model centered on the Merkel cell. Merkel cells were first studied in the late 19th century, but they are still not fully understood [15]. They have been suggested to be responsible for various growth formations such as hair follicles, bringing nerves to the skin, and Kücken and
Champod suggest that they control the formation of ridge patterns on the skin [15]. This theory has been suggested by several other researches dating back to the mid 1980’s. It is believed that after the stress caused by skin growth and volar pad shrinkage and that the Merkel cells form in a pattern based on the basic ridges formed from these pressures [15]. The Merkel cells are then responsible for the formation of the primary ridge patterns. For the model described by Kücken and Champod, each Merkel cell, \(i\), is described by its position \(\vec{x}_i\) [15]. This position for each Merkel cell is initially random and moves according to stress tensor fields and aligns along the lines of smallest stress [15]. Each Merkel cell interacts with another cell in one of two ways, attraction, \(\vec{F}_a\), or repulsion, \(\vec{F}_r\) [15]. Thus the movement of a given Merkel cell is determined by the sum of all attractive and repulsive forces from other Merkel cells in Equation 7, with \(T\) as the tensor field [15].

\[
\rho \vec{x}_i = \sum_{j \neq i} \vec{F}_a (\vec{x}_ij T) + \vec{F}_r (x_{ij})
\]  

(7)

The attractive and repulsive forces respectively are described by Equation 8 and Equation 9, respectively, with \(\vec{s}\) and \(\vec{l}\) being unit vectors describing the directions of the smallest and largest compression in the tensor field [15].

\[
\vec{F}_a (\vec{r}, T) = \gamma r e^{-e r} \left( \chi (\vec{s} \cdot \vec{r}) \vec{s} + (\vec{l} \cdot \vec{r}) \vec{l} \right)
\]  

(8)

\[
\vec{F}_r (\vec{r}) = (\alpha r^2 + \beta) (e^{-e r}) \vec{r}
\]  

(9)

\(\chi\) determines the direction of the attraction and where the Merkel cells align to form the ridge patterns and the other variables are other parameters set for the model [15]. For the beginning of the simulation, only the repulsive forces are described and attractive forces are added slowly as the simulation progresses, beginning in large compression stress areas and slowly adding in areas with smaller stress [15]. There are also spring forces set at the boundary of the simulation to keep the Merkel cells contained [15]. Various assumptions and adjustments are added to deal with the large number of agents being simulated [15].

Tensor fields obtained from real world fingerprints are used for the most accurate result. These simulations show Merkel cells aligning in a way that generally describes the ridge pattern of the fingerprint, no matter what the initial Merkel cell distribution is [15]. However, small differences are found when it comes to more detailed ridge patterns and minutiae which are due to the profound effect even single cells can have on these formations [15]. After further study, it was found that these differences were very similar to the differences found in the fingerprints of identical twins, which supports the model of the Merkel cell as the agent to cause pattern formation [15]. Several problems are found in the model, such as open areas without minutiae and minutiae ratio inaccuracies. While the model is not perfect, it better described the formation of friction ridge skin than any other current model [15]. These problems with the model
can probably be reduced by better description of initial conditions and better equation modeling of the Merkel cells [15]. Kucken and Champod believe that the Merkel cell agent based model is not inaccurate, but incomplete [15]. Their findings lend weight to the hypothesis that Merkel cells play a very important role in ridge pattern formation.

Conclusions

Altogether the study of fingerprints and the creation of synthetic fingerprints is an interesting field of study. Using the statistical information about the location of minutiae, synthetically generated fingerprints may be tested to determine the realistic qualities of these prints. In terms of actually generating fingerprints, there are some promising methods. The use of pre-existing fingerprints to generate false prints is a novel idea, but it has limitations in that the created prints will be highly related to the master fingerprint. These prints also only generate prints similar to the master fingerprint. In terms of using the theory of elasticity and agent formation, these models may be promising for the creation of fingerprints without master fingerprints to generate the pattern. These theories assist in the explanation of the formation of friction ridge skin; thus if mathematical models could be created to simulate the process of fingerprint formation, it would be possible to create a database of fingerprints to test fingerprint matching algorithms. Ultimately, the study and creation of synthetic fingerprints could lead to improved fingerprint security systems and better fingerprint identification methods.

References


