Intermittency

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What is Intermittency?

Systems which are stable, then witness a burst of chaos, then return to stability, then witness another chaotic burst, etc. etc...

Three fundamental types:

Pomeau Manneville: Bifurcations

Crisis-Induced: From one chaotic attractor to another

On-Off: From a stable chaotic attractor, unstable orbits shot into hyperplane
What is Intermittency?

These “chaotic bursts” are observed in models of turbulence, bipolar disorder, and even our favorite map: \( g(x) = ax(1-x) \).
Pomeau Manneville Intermittency

Each type of PM Intermittency is related to some kind of bifurcation.

I) A saddle node bifurcation - Annihilation of fixed points

II) A subcritical Hopf bifurcation - Previously stable limit cycle becomes repelling

III) An inverse period doubling bifurcation - Period k, k/2, k/4….

For the logistic map, we have a type I transition to intermittency.
Intermittency in the Logistic Map

These white spaces, the “periodic windows” wherein the map suddenly displays periodic behavior, appear in the region of $3.566 < a < 3.828$ for $g(x) = ax(1-x)$.

At $a = 3.828$, a saddle node bifurcation occurs and the map loses intermittency in the periodic window up to $a = 4$. 
Intermittency in the Logistic Map
Intermittency in the Logistic Map

After a certain number of iterations, the orbit leaves this window, becoming chaotic, only to return again to this laminar phase before leaving again.

This return is “injection”.

Simulation: https://personalpages.manchester.ac.uk/staff/ yanghong.huang/teaching/MATH4041/DS/intermittency.html
Intermittency in the Lorenz System

Another example of type I Pomeau-Manneville intermittency.

The system is a simplified model of atmospheric convection.

\[
\begin{align*}
\frac{dx}{dt} &= k(y-x), \\
\frac{dy}{dt} &= -xz + rx - y, \\
\frac{dz}{dt} &= xy - bz
\end{align*}
\]

Where \( k = 10 \), \( b = 8/3 \), \( r \) are parameters. In this system, variation of the parameter \( r \) leads to intermittent phenomenon.

For \( 166 < r < 166.07 \), the system is in a stable limit cycle. Yet for \( r = 166.07 \), the map transitions from a stable limit cycle into a strange attractor at a saddle node bifurcation.
Intermittency in the Lorenz System

Laminar phases decrease as $r$ gets further away from $r_t$. 
Laminar phase length

The length of the average laminar phase increases as the parameter $a$ approaches 3.828.

It was shown by Saramah et. al that for the number of iterations $N$ that the orbit spends in the laminar phase,

$$N \propto \varepsilon^{-1/2}$$

Where epsilon is the difference between the critical bifurcation and the current parameter.
Laminar phase length per phase

The exact length of each individual laminar phase for a given map for any initial point is unpredictable.

This is because it is impossible to predict exactly when or at what exact value the chaotic orbit will return to the laminar phase.

The number of iterations for which the orbit remains in the laminar phase depends very sensitively on how closely it was mapped to the periodic orbit when it was injected from the chaotic phase.
Crisis-Induced Intermittency

Two or more chaotic attractors cross the boundary of each others’ basin of attraction. Then an orbit can swap between the two (or more!) attractors.

Here we see the driven Duffing equation oscillate between potential wells.
Bipolar Disorder: Crisis Induced Intermittency

In healthy subjects, mood is correlated with one strange attractor - in Bipolar Disorder, disease replaces this strange attractor with two abnormal attractors.
Duffing Equation - Crisis Induced Intermittency

As parameter varies, the coexisting chaotic attractors transition from being independent to crossing each others’ basins.

The pattern here is Chaos1 => Chaos 2 => Chaos1…

Though more than two attractors may be involved. Then the movement from attractor to attractor need not be periodic.
History of Intermittency

Much like any mathematical concept, Intermittency has gone through several models. Some of these models are based on dissipation and velocity. A couple of note are the shell and Kraichman models. **<maybe mention in turbulence slide>**

The word intermittence may even derive from its use in fluid mechanics. “In fluid turbulence, the term was introduced to describe signals from probes in fluids that alternated between flat portions and bursty ones, interpreted as laminar and turbulent states of the fluid.” (Platt et al p279)

Applications with henon, lorentz, etc.
On-Off Intermittency

“On” state

- Burst
- Quickly returning to “off”

“Off” state

- Nearly constant
- Long
- Laminar phases

So, how is this different than intermittency in general?
Another kind, on-off intermittency, occurs when a previously transversally stable chaotic attractor with dimension less than the embedding space begins to lose stability. Near unstable orbits within the attractor orbits can escape into the surrounding space, producing a temporary burst before returning to the attractor.

On-Off Intermittency
On-Off Intermittency: Simplified Model

Pics to be added later of different time series representations

ex) logistic map intermittency for $a = 2.8$

The second time series displays the hallmark of on-off intermittent behavior — long periods of nearly constant signal interrupted by short-lived larger amplitude bursts.

Logistic map is not on-off.
On-Off Intermittency

On-Off Intermittency stems from the repeated variation of a dynamical variable through a bifurcation point of another. The response of this second variable is where the intermittent signal is derived from. This can be studied using a simple one-dimensional map

\[ y_{n+1} = z_n f(y_n) \quad \text{with} \quad f(0) = 0, \quad r f(y) / 0 \text{ fix later} \quad z_n \text{ is based off of the chaotic or random density function } p_z(x) \text{ list all conditions} \]

This map can take different forms, ie the logistic function, under the right circumstances
Turbulence: What is it?

If you have ever been in an airplane, you probably have an idea of what turbulence is. Simply put, it is a disruption of a space that makes most fear for their lives. In relation to chaos and more specifically intermittency, turbulence is a physical application of chaos. Irregular fluctuations in temperature, pressure, and other factors lead to things getting shaken up. These chaotic bursts are essentially the second phase of intermittency. Turbulence can present itself in any fluid, not just in the sky surrounded by an airplane. This sounds a lot like what we have studied about chaos in general.
Power Law Distribution
Sources

http://shodhganga.inflibnet.ac.in/bitstream/10603/88951/9/09_chapter%203.pdf

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