Section 4.6

14. \( f(x) = 4x^4 - 15x^2 - x - 2 \)
   
   \( x - 2 \Rightarrow \)
   
   \( x = 2 \)
   
   \( f(2) = 4(2)^4 - 15(2)^2 - 2 \)
   
   \( f(2) = 64 - 60 - 2 \)
   
   \( f(2) = 64 - 62 \)
   
   \( f(2) = 2 \)
   
   Since \( f(2) \neq 0 \), \( x - 2 \) is not a factor.

57. \( f(x) = 0 = x^4 - x^3 + 2x^2 - 4x - 8 \)

   \( x = \frac{\pm1, \pm2, \pm4, \pm8}{\pm1} = \pm1, \pm2, \pm4, \pm8 \)

\( f(x) = (x-1)(x+2)(x^2+4) \)

\( x = \pm\sqrt{2} \Rightarrow x \pm 2 \Rightarrow x = \pm\sqrt{2} \)

\( x = -2 \Rightarrow x = 0 \)

\( x = 1 \Rightarrow x = 0 \)

\( x = 0 \Rightarrow x = 0 \)

\( x = 1 \Rightarrow x = 0 \)

\( x = 2 \Rightarrow x = -2 \)

\( \begin{array}{cccc}
-1 & 1 & 1 & 2 \\
0 & 2 & -4 & -8 \\
2 & 1 & -2 & 4 & -8 \\
\hline
2 & 0 & 8 & 0 \\
1 & 0 & 4 & 0 \\
\end{array} \)
1. \( 2x^4 + x^3 - 24x^2 + 20x + 16 = 0 \)
   \[ \frac{p}{b} = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16 \]
   \[ \frac{p}{b} = \pm \frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8, \pm 16 \]

2. \( x = 2 \) or \( 2 \)
   \( x^2 - \frac{1}{2} \)
   \( x = -y \)

3. \[
\begin{array}{ccccccc}
2 & 2 & 1 & -24 & 20 & 16 \\
& 4 & 10 & -28 & -16 \\
2 & 2 & 5 & -14 & -8 & 10 \\
& 4 & 18 & 8 \\
& -4 & 2 & 9 & 4 & 10 \\
& -8 & -4 \\
2 & 1 & 10 \\
\end{array}
\]

\( 2x = 1 \)
\( x = 2 \)
\( x = -y \)

\( x = -\frac{1}{2} \)
\( x - 2 = 0 \)
\( x + y = 0 \)

\( f(x) = 2(x + \frac{1}{2})(x-2)^2(x+y) \)
Section 4.7

18. Degree 4: \( 2x^4 + 3i = 1 + 2i \)
\[
\begin{align*}
\text{roots:} & = i, -i, 1+2i, 1-2i \\
\text{factors:} & = (x-i)(x+i)(x-(1+2i))(x-(1-2i)) \\
& = (x^2+1)(x-1-2i)(x-1+2i) \\
& = (x^2+1)((x-1)^2 - 4i^2) \\
& = (x^2+1)(x^2-2x+1+4) \\
& = (x^2+1)(x^2-2x+5) \\
& = x^4 - 2x^3 + 5x^2 + x^2 - 2x + 5 \\
& = x^4 - 2x^3 + 6x^2 - 2x + 5
\end{align*}
\]

26. \( h(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18 \)
\[
\begin{align*}
\text{roots:} & = 3i, -3i \\
\text{factors:} & = \frac{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18}{\pm 1, \pm 3} \\
& = \pm \frac{1}{3}, \pm \frac{2}{3}, \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18 \\
\text{roots:} & = 3i \text{ also } -3i \\
\text{factors:} & = (x-3i)(x+3i) = (x^2-9i^2) \\
& = x^2 + 9
\end{align*}
\]
\[ x^2 + 10x + 9 \]

\[ 3x^2 + 5x - 2 \]

\[ 3x^4 + 5x^3 + 25x^2 + 45x - 18 \]

\( \text{I} \) \[ 3x^4 + 0x^3 + 27x^2 \]

\[ 5x^3 - 2x^2 + 45x - 18 \]

\( \text{II} \) \[ 5x^3 + 0x^2 + 45x \]

\[ -2x^2 + 0x - 18 \]

\( \text{III} \) \[ 2x^2 + 0x + 8 \]

\[ -2 \]

\[ 3 \]

\[ 5 \]

\[ -2 \]

\[ -6 \]

\[ 2 \]

\[ \frac{1}{3} \]

\[ 3 \]

\[ 3 \]

\[ 10 \]

\[ f(x) = 3(x^2+9)(x+2)(x-\frac{1}{3}) \]

37. \[ f(x) = x^4 + 2x^3 + 22x^2 + 50x - 75 \]

\[ f' = \pm 1, \pm 3, \pm 5, \pm (15), \pm 25, \pm 75 \]

\[ -3 \]

\[ 1 \]

\[ 3 \]

\[ -3 \]

\[ 3 \]

\[ 25 \]

\[ 75 \]

\[ 0 \]

\[ 25 \]

\[ 10 \]

\[ x^2 + 25 = 0 \]

\[ x^2 = -25 \]

\[ x = \pm \sqrt{25} \]

\[ x = \pm 5i \]

\[ x = 1, -3, \pm 5i, -5i \]
Homework

Section 5.1

39. a) \( f \circ g \)  b) \( g \circ f \)  c) \( f \circ f \)  d) \( g \circ g \)

Find each of the following, state the domain

\[
\begin{align*}
  f(x) &= x^2 + 1 \\
  g(x) &= \sqrt{x-1} \\
  &\text{domain } (-\infty, \infty) \quad x - 1 > 0 \quad x > 1 \quad [1, \infty)
\end{align*}
\]

a) \( (f \circ g)(x) \)

\[
(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x-1}\right)
\]

\[
= \left(\sqrt{x-1}\right)^2 + 1 = x - 1 + 1 = x
\]

\text{domain } [1, \infty)

b) \( (g \circ f)(x) \)

\[
(g \circ f)(x) = g(f(x)) = g(x^2 + 1)
\]

\[
= g\left(\sqrt{x^2 + 1}\right)
\]

\[
= \frac{1}{\sqrt{x^2 + 1}} - 1 = 0
\]

\[
\frac{\sqrt{x^2} - 1}{1} = \frac{x}{1} - 1 = 0
\]

\[
\sqrt{x^2} - 1 = 0
\]

\[
x - 1 = 0
\]

\[
x = 1
\]

\text{domain } (-\infty, \infty)

\[
\frac{\sqrt{x^2} - 1}{1} = \frac{x - 1}{1} = 0
\]

\[
x = 1
\]

\text{domain } (-\infty, \infty)

\[
\frac{\sqrt{x^2} - 1}{1} = \frac{x - 1}{1} = 0
\]

\[
x = 2
\]

\text{domain } (-\infty, 2) \cup (2, \infty)

\[
\text{error}
\]

\[
0 \\
3 \frac{\sqrt{5} - 1}{1}
\]

\[
2 \quad \text{error}
\]
52. \( H(x) = (1 + x^2)^3 \)
\[
(f \circ g)(x) = H
\]
\[
g(x) = 1 + x^2
\]
\[
f(x) = x^3
\]

65. \( p = -\frac{1}{4} x + 100 \quad 0 \leq x \leq 400 \)
\[
c(x) = \frac{\sqrt{x}}{25} + 1000
\]

\[\text{Find: } c(p)\]

\[\text{Solution}\]

\[
p = -\frac{1}{4} (0) + 100
\]
\[
p = 100
\]
\[
p = -\frac{1}{4} \cdot 400 + 100
\]
\[
p = -100 + 100
\]
\[
p = 0
\]
\[
p = -\frac{1}{4} x + 100
\]
\[
p - 100 = -\frac{1}{4} x
\]
\[
-4 (p - 100) = x
\]
\[
x = -4p + 400
\]

\[c(p) = \frac{\sqrt{400 - 4p}}{25} + 1000\]
46. \( f(x) = x^2 + 9 \) \( x \geq 0 \) domain \( \mathbb{R} \) range \( [9, \infty) \)

\( y = x^2 + 9 \)
\[ \frac{y - 9}{x} = x \]
\[ \sqrt{y - 9} = x \]

\( \sqrt{x - 9} = y \)
\[ \sqrt{x - 9} = f'(x) \) domain \( [9, \infty) \) range \( [0, \infty) \)

57. \( f(x) = \frac{3x + 4}{2x - 3} \) domain \( (-\infty, 3/2) \cup (3/2, \infty) \)

\[ 2x - 3 = 0 \]
\[ 2x = 3 \]
\[ x = \frac{3}{2} \]

\( \lim_{x \to 3/2} f(x) = \lim_{x \to 3/2} \frac{3x + 4}{2x - 3} \]
\[ = \lim_{x \to 3/2} \frac{3 + \frac{4}{x}}{2 - \frac{3}{x}} \]
\[ = \frac{3}{2} + \frac{4}{\frac{3}{2}} \]
\[ = \frac{3}{2} \]

H. A. \( y = \frac{3}{2} \)
\[ y = \frac{3x+y}{2x-3} \]

\[ (2x-3)y = 3x+y \]

\[ 2y-3y = 4+3y \]

\[ x(2y-3) = 4+3y \]

\[ x = \frac{4+3y}{2y-3} \]

\[ y = \frac{4+3x}{2x-3} \]

\[ f^{-1}(x) = \frac{4+3x}{2x-3} \]

Domain: \((-\infty, \frac{3}{2}) \cup \left(\frac{3}{2}, \infty\right)\)

Range: \((-\infty, \frac{3}{2}) \cup \left(\frac{3}{2}, \infty\right)\)
5.3 \( \text{domain } (0, \infty) \text{ range } (0, \infty) \)

49 \[
\begin{array}{c|c|c|c}
 x & e^x & e^{-x} & -e^{-x} \\
-1 & 0.37 & -1 & 2.72 \\
0 & 1 & 0 & 1 \\
1 & 2.72 & 1 & 0.37 \\
\end{array}
\]

\[y = e^x \]
\[y = 5 - e^{-x} \]
\[y = 5 - e^{-x} - 1 \]
\[y = 4 \]
\[y = 4.63 \]

\[y = e^x \text{ domain } (0, \infty) \text{ range } (0, 5) \]

59 \[
2^x \cdot 8^{-x} = 4^x \\
2^x \cdot (2^3)^{-x} = (2^2)^x \\
2^x \cdot 2^{-3x} = 2^{2x} \\
2^{-2x} = 2^{2x} \\
-2x = 2x \\
0 = 4x \\
0 = x
\]
79. \( D(h) = \frac{5e^{-0.4h}}{h} \)

\( h = \text{min} \)

\( D = \text{milligrams} \)

a) \( D(1) = \frac{5e^{-0.4 \cdot 1}}{1} = 3.3516 \text{ milligrams} \)

b) \( D(0.1) = \frac{5e^{-0.4 \cdot 0.1}}{0.1} \approx 0.4536 \)
52. \( g(x) = \ln \left( \frac{1}{x-5} \right) \)

\[ \frac{1}{x-5} > 0 \]
\[ x-5 = 0 \]
\[ x = 5 \]
\[ (-\infty, 5) \quad (5, \infty) \]

\( \frac{1}{4-5} \quad \frac{1}{6-5} \)
\[ \ln \frac{1}{4-5} \quad \ln \frac{1}{6-5} \]
\[ \ln \left( \frac{1}{4-5} \right) = 0 \quad \ln 1 = 0 \]

Domain: \((5, \infty)\)

89. \( h(x) = \log (x+2) + 3 \)

\[ y - 3 = \log (x+2) \]
\[ x + 2 = 10^{(y-3)} \]
\[ x = -2 + 10^{(y-3)} \]

Domain: \(x+2 > 0\)

\[ x > -2 \quad (-2, \infty) \]

Vertical Asymptote: \( x = -2 \)

Range: \((-\infty, \infty)\)
10.2
\[ \log_6 36 = 5x + 3 \]
\[ 36 = 6^{5x+3} \]
\[ (6)^2 = 6^{5x+3} \]
\[ 2 = 6^{5x+3} \]
\[ 5x + 3 = -1 \]
\[ x = -\frac{4}{5} \]

Section 5.5

10. \( \ln e^{\sqrt{2}} = \sqrt{2} \). Let \( e^y = 1 \),
\[ \sqrt{2} = \sqrt{2} \]

48. \( \ln \left[ \frac{(x-4)^2}{x^2-1} \right]^{\frac{2}{3}} \) \( x > 4 \)

\[ \frac{2}{3} \ln (x-4)^2 - \frac{2}{3} \ln (x^2-1) \]

57. \( \ln \left( \frac{x}{x+1} \right) + \ln \left( \frac{x+1}{x} \right) - \ln (x^2-1) \)

\[ = \ln \left( \frac{x(x+1)}{(x-1)(x+1)} \right) - \ln (x^2-1) \]

\[ = \ln \left( \frac{x+1}{(x-1)(x+1)} \right) - \ln (x^2-1) \]

\[ = \ln \left( \frac{x+1}{(x-1)(x+1)} \right) - \ln (x^2-1) \]

\[ = \ln \left( \frac{1}{(x-1)^2} \right) \]
8. \( 2 \log_3 (x+4) - \log_3 9 = 2 \)
\[ \log_3 \left( \frac{(x+4)^2}{9} \right) = 2 \]
\[ 3 \log_3 \frac{(x+4)^2}{9} = 3^2 \]
\[ 9 \cdot \frac{(x+4)^2}{9} = 9 \cdot 9 \]
\[ (x+4)^2 = 81 \]
\[ \sqrt{(x+4)^2} = \pm 9 \]
\[ x+4 = \pm 9 \]
\[ x+4 = 9 \quad x+4 = -9 \]
\[ x = 5 \quad x = -13 \]

\( x = -13 \) is not possible because the domain of the logarithmic function requires \( x+4 > 0 \).

Domain: \( x+4 > 0 \)
\[ x > -4 \]
\[ x = 5 \] only answers.

34. \( \log_4 (x^2 - 9) - \log_4 (x+3) = 3 \)
\[ \log_4 \frac{x^2 - 9}{x+3} = 3 \]
\[ 4 \log_4 (x^2 - 9) = 4^3 \]
\[ x^2 - 9 = 64 \]
\[ x = 85 \quad x = -85 \]
\[(x + 3)(x - 3) > 0\]

\[
\begin{array}{c|ccc}
\infty & -3 & 3 & -3 \\
\hline
- & - & + & + \\
\end{array}
\]

\[
\begin{array}{c|ccc}
-3 & - & - & + \\
\hline
+ & + & - & + \\
\end{array}
\]

\[x = 0.7 \text{ is a solution}\]

57. \[e^x + 10x = 4\]

\[\sqrt[4]{\left(1.3153, 4\right)}\]