6.3 Working with Normally Distributed Variables

To Determine a Percentage or Probability for a Normally Distributed Variable

Step 1: Sketch the normal curve associated with the variable
Step 2: Shade the region of interest and mark its delimiting x-value(s).
Step 3: Find the z-score(s) for the delimiting x-value(s) found in Step 2.
Step 4: Use Table II to find the area under the standard normal curve delimited by the z-score(s) found in Step 3.

Examples
A variable is normally distributed with mean 68 and standard deviation 10. Find the percentage of all possible values of the variable that
a. lie between 73 and 80
b. are at least 75
c. are at most 90

The 68.26-95.44-99.74 Rule
Any normally distributed variable has the following properties

Property 1: 68.26% of all possible observations lie within one standard deviation to either side of the mean, that is, between $\mu - \sigma$ and $\mu + \sigma$

Property 2: 95.44% of all possible observations lie within two standard deviation to either side of the mean, that is, between $\mu - 2\sigma$ and $\mu + 2\sigma$

Property 3: 99.74% of all possible observations lie within three standard deviations to either side of the mean, that is, between $\mu - 3\sigma$ and $\mu + 3\sigma$
To Determine the Observations Corresponding to a Specified Percentage or Probability for a Normally Distributed Variable

**Step 1:** Sketch the normal curve associated with the variable

**Step 2:** Shade the region of interest.

**Step 3:** Use Table II to determine the z-score(s) for the delimiting region found in Step 2.

**Step 4:** Find the x-value(s) having the z-scores(s) found in Step 3.

**Examples**
A variable is normally distributed with mean 68 and standard deviation 10.
   a. Determine and interpret the quartiles of the variable
   b. Obtain and interpret the 99\textsuperscript{th} percentile
   c. Find the value that 85\% of all possible values of the variable exceed.
   d. Find the two values that divide the area under the corresponding normal curve into a middle area of 0.90 and two outside areas of 0.05. Interpret your answer.

**Examples**
According to the *National Health and Nutritional Examination Survey*, published by the national Center for Health Statistics, the serum (noncellular portion of blood) total cholesterol level of U.S. females 20 years old or older is normally distributed with a mean of 206 mg/dL (milligrams per deciliter) and a standard deviation of 44.7 mg/dL.
   a. Determine the percentage of U.S. females 20 years old or older who have a serum total cholesterol level between 150 mg/dL and 250 mg/dL.
   b. Determine the percentage of U.S. females 20 years old or older who have a serum total cholesterol level below 220 mg/dL.
   c. Obtain and interpret the quartiles for serum total cholesterol level of U.S. females 20 years old or older.
   d. Find and interpret the fourth decile for serum total cholesterol level of U.S. females 20 years old or older.