Section 7.5
Conditional Probabilities and Independence

Contingency Tables

A contingency table is a table for bivariate data. It can be used to show the joint probabilities such as $P(A \cap B)$ and the conditional probabilities such as $P(A|B)$ or $P(B|A)$

Example 1

A survey was taken. The subjects were sorted by income level and were either labeled as "stressed" or "not stressed"

	Income			
	Low	Medium	High	**Total**
Stressed	**526**	**274**	**216**	**1016**
Not Stressed	**1954**	**1680**	**1899**	**5533**
Total	**2480**	**1954**	**2115**	**6549**

The row totals and column totals are the **individual probabilities** divided by the grand total so the $P(Low\ Income)$ would be

$$P(Low\ Income) = \frac{2480}{6549}$$

The information inside the heavily boarded cells is **joint information** so the number of people labeled as Stressed **and** Low Income would be 526.

The $P(Stressed \text{ and } Low\ Income) = \frac{526}{6549}$

<u>Conditional probability:</u> the probability of an event occurring given that another event has already occurred.

The **<u>conditional probability</u>** of being **<u>Stressed given that the person is Low Income</u>** would be given by the following formula.

The

$$P(Stressed \mid Low\ Income) = \frac{P(Stressed \cap Low\ Income)}{P(Low\ Income)}$$

$$P(Stressed \mid Low\ Income) = \frac{526/6549}{2480/6549} = \frac{526}{2480}$$

Notice that the grand total cancels out so it is not even needed in the equation. Just the joint counts for **<u>Stressed and Low Income</u>** and the individual count for **<u>Low Income</u>**

This conditional probability could also have been written as "The low income person is stressed"

<u>Example 2</u>

Find the following probabilities:

A. P (*S*)

B. P (*S* U *L*)

C. $P(Low\ Income \mid Stressed)$

D. Compare $P(Stressed \mid Low\ Income)$ and $P(Stressed \mid High\ Income)$

E. The middle income person is not stressed.

Dependent events: Is when one event having occurred changes the probability of a second event occurring.

Independent events: Is when one event having occurred does not change the probability of a second event occurring.

Rules about independent and dependent data

1. If two events A and B are independent, then
   $P(A \cap B) = P(A) \cdot P(B)$

2. For any two events independent or dependent
   $P(A \cap B) = P(A) \cdot P(B|A)$ or
   $P(A \cap B) = P(B) \cdot P(A|B)$

Example 3

Refer to Example 1: Is being stressed independent of being low income?

46. In 2004, the probability that a person in the United States would declare personal bankruptcy was 0.006. The probability that a person in the United States would declare personal bankruptcy and had recently over spent credit cards was 0.0004. What was the probability that a person had recently overspent credit cards given that he had declared personal bankruptcy?

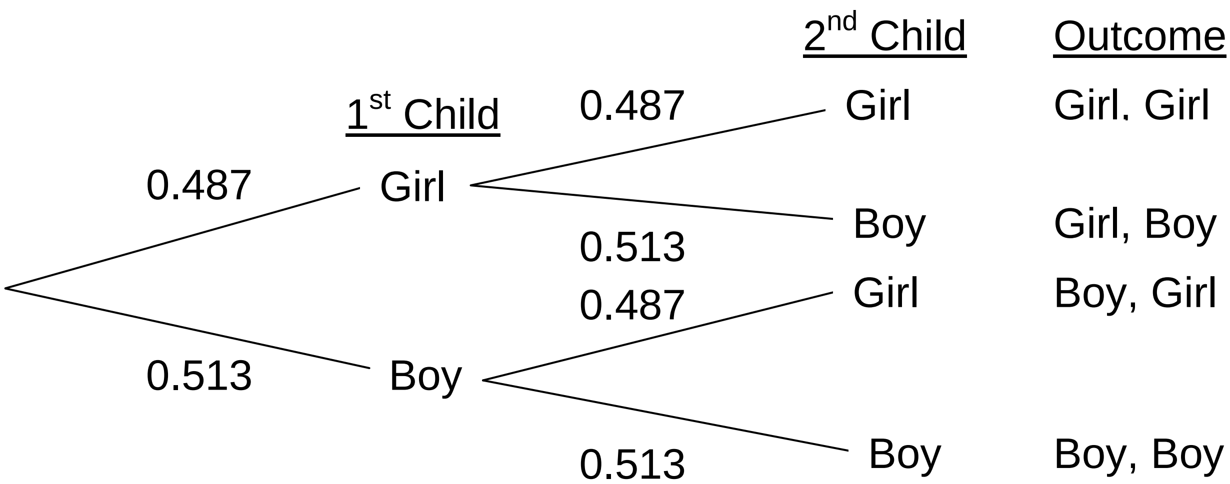
Let's move on to Tree Diagrams or probability trees (a personal favorite)

Probability Trees

Probability tree is a convenient way to break a problem down into parts and organize information.

Example 4

A family has two children. The probability of having a girl is 0.487



Two rules with Tree diagrams

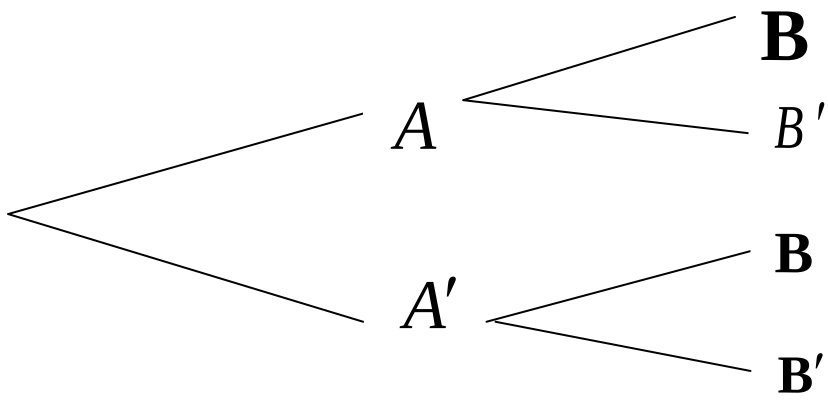
1. When you are traveling along a branch you multiply the probabilities.
2. When you go from branch to branch you add.

Find the following probabilities:

1. $P(Girl1 \cap Girl2)$
2. $P(exactly 1 Girl)$
3. $P(Girl')$

The reason to do this example is to help us to relate tree diagrams to a basic situation.

<u>Conditional probability:</u> the probability of an event occurring given that another event has already occurred.



$P(B|A)$ reads as the probability that Event B occurs given that Event A has already occurred.

$\Pr(B|A) = \dfrac{\Pr(B \cap A)}{\Pr(A)}$ provided $\Pr(A) > 0$

Also the following equations are also true

$\Pr(A|B) \cdot \Pr(B) = \Pr(A \text{ and } B)$ and

$\Pr(B|A) \cdot \Pr(A) = \Pr(A \text{ and } B)$

Rule for Total Probability

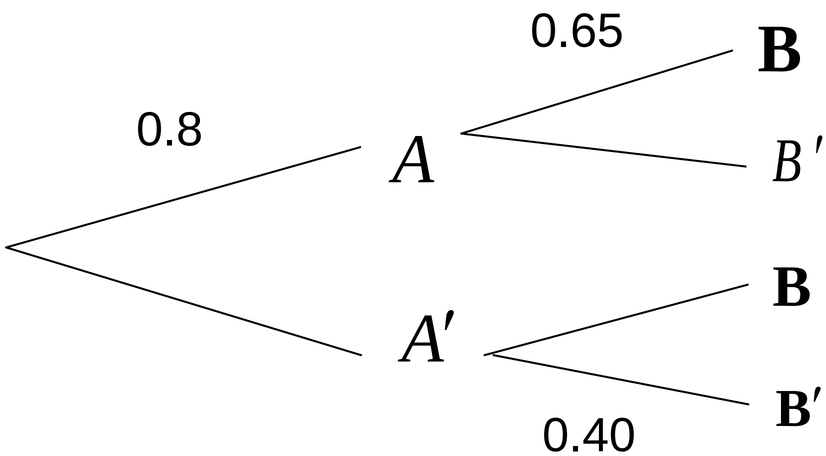
For any two events A and B

$P(B) = P(B|A) \cdot P(A) + \Pr(B|A') \cdot \Pr(A')$

Example 5

Given: A is chocolate Halloween candy, B is probability that the candy has nuts.

1. Fill in the tree diagram and determine the outcomes.
2. Find the probability that the candy has nuts.



66. It appears that there is only a one in five chance that you will be able to take your spring break vacation to the Greek Islands otherwise you will stay home. If you are lucky enough to go, you will visit either Corfu (20% chance) or Rhodes. On Rhodes, there is a 20% chance of meeting a tall dark stranger, while on Corfu, there is no such chance.