Section 15.4 Constrained Maxima and Minima and Applications

In Brief Calculus we placed constraints on our variables in max/min problems. We will be doing this same with our functions of many variables. The two methods that we will be studying are

- The substitution method.
- The Lagrange multiplier method.

**Substitution Method**
1. Determine the objective function. This is the function to be maximized or minimized.
2. Determine the constraints. These are the limitations that are placed on the objective function.
3. Solve the constraint equation for one of the variables and then substitute into the objective function.
4. Minimize/maximize the resulting function of two variables. (We look for the critical points.
5. Verify that these points are in agreement with the domain.
6. Apply the second derivative test to determine the value of H.
7. Solve for the remaining variable.
8. Solve for the corresponding value of the objective function.
Example 1 Solve the given optimization problem by using substitution pg. 1126

3. Find the maximum value of \( f(x, y, z) = 1 - x^2 - x - y^2 + y - z^2 + z \) subject to \( 3x = y \). Also find the corresponding point(s) \( (x, y, z) \)

5. Minimize \( S = xy + 4xz + 2yz \) subject to \( x, y, z = 1 \) with \( x > 0, y > 0, z > 0 \)

When it is impossible to solve a constraint equation for one of the variables, we can use Lagrange Multipliers

**Lagrange Multiplier Method for locating relative extrema**

To locate the candidates for relative extrema of a function \( f(x, y,\ldots) \) subject to the constraint \( g(x, y,\ldots) = 0 \)

1. Construct the **Lagrangian function**
   \[
   L(x, y,\ldots) = f(x, y,\ldots) - \lambda g(x, y,\ldots)
   \]
   Where \( \lambda \) is a new unknown called a **Lagrange multiplier**.

2. The candidates for the relative extrema occur at the critical points of \( L(x, y,\ldots) \). To find them all, set the partial derivatives of \( L(x, y,\ldots) \) equal to zero and solve the resulting system, together with the constraint equation \( g(x, y,\ldots) = 0 \) for the unknowns \( x, y,\ldots\lambda \)

3. The points \( (x, y,\ldots) \) that occur in solutions are then the candidates for the relative extrema of \( f \) subject to \( g = 0 \)
Example 2
Problems 8 and 12 pg 1126 in the book.

8. Find the maximum value of \( f(x, y) = xy \) subject to \( 3x + y = 60 \).
Also find the corresponding point(s) \((x, y)\)

12. Find the minimum value of \( f(x, y) = x^2 + y^2 \) subject to \( xy^2 = 16 \).
Also find the corresponding point(s) \((x, y)\)