Section 9.1 Dichotomous Observations

**Dichotomous observations**: when only two types of observations exist.

What kind of distribution has dichotomous observations?
The binomial distribution has two types of categories. One category is considered a “success” and the other category is considered a “failure”

We can define the proportion of each category

\[ p \] is the population proportion of one category

\[ \hat{p} \] is the sample proportion of one category

In Chapter 6 we used sampling distributions to show how sample means could vary from the population mean.
Now we will use a sampling distribution for a sample proportion and determine how that would fit with a dichotomous categorical variable.
The Wilson Adjusted Sample proportion $\tilde{p}$ is used in this book

$$\tilde{p} = \frac{y + 2}{n + 4}$$

where $y$ is the number of observations in the sample with the particular attribute and $n$ is the number of observations in the sample.

Notice that this equation increases the number of observations with the particular attribute by 2 and the total number of observations by 4.

Because of this change $\tilde{p}$ is closer to $\frac{1}{2}$

Example 1 (Problem 9.1.8)

In a certain community 17% of the soda dispensers are contaminated. Suppose a random sample of five dispensers are chosen at random, and contamination is observed. Let $\tilde{p}$ represent the Wilson Adjusted Sample proportion of contaminated dispensers.

a. Compute the sampling distribution Note that you will need to use the Binomial Distribution
Equation to complete this problem. Refer to Section 3.6

\[ \Pr(Y = j) = \Pr(j \text{ successes}) = \binom{n}{j} p^j (1 - p)^{n-j} \]

b. What is the probability that at most 2 dispensers are contaminated?

c. What is the probability that the sampling error will be within 5% of \( p \) (inclusive)?