Volume by cylindrical shells

Ex 2: Rotate the region bounded by $x^2 - y^2 = 9$ and $x = 5$ about the line $y = 6$. Use cylindrical shell method to set up the integral for the volume of the solid of revolution obtained.

\[5^2 - y^2 = 9\]
\[y^2 = 25 - 9 = 16\]
\[y = \pm 4\]

\[x^2 - y^2 = 9\]
\[x^2 = y^2 + 9\]
\[x = \pm \sqrt{y^2 + 9}\]

\[V_i = 2\pi rh \cdot \Delta y = 2\pi (6 - y_i) \cdot (5 - \sqrt{y_i^2 + 9}) \Delta y\]

\[V \approx \sum_{i=1}^{n} 2\pi (6 - y_i) \cdot (5 - \sqrt{y_i^2 + 9}) \Delta y\]

\[V = \int_{-4}^{4} 2\pi (6-y)(5-\sqrt{y^2+9}) \, dy\]