Example: Find the area enclosed by the curve $r = 1$ and $r^2 = 25\theta$.  

Solution: 

Area of $A_1$ is given by: 

$$A_1 = \frac{1}{2} \int_0^{\pi/12} 25\theta \, d\theta$$ 

$$= \left[ \frac{1}{2} \theta^2 \right]_0^{\pi/12}$$ 

$$= \frac{1}{2} \left( \frac{\pi^2}{144} \right) = \frac{\pi^2}{288}$$ 

Area of $A_2$ is given by: 

$$A_2 = \frac{1}{2} \int_{\pi/12}^{\pi/4} 1^2 \, d\theta$$ 

$$= \left[ \frac{\theta}{2} \right]_{\pi/12}^{\pi/4}$$ 

$$= \frac{\pi}{24}$$ 

Area is: 

$$A = 4 \left( A_1 + A_2 \right)$$ 

$$= 4 \left( \frac{\pi^2}{288} + \frac{\pi}{24} \right)$$ 

$$= \frac{\pi^2}{72} + \frac{\pi}{6}$$ 

$$= 2 - \sqrt{3} + \frac{\pi}{3}$$