Volume of a solid using slicing

**\( \text{EX4} \)**

Find the volume of the monument that has height 20 m and its horizontal cross sections at a distance \( x \) meters from the top are equilateral triangles with sides \( \frac{x}{4} \) meters.

Idea: Visualize the monument lying side ways.

Slice the solid in perpendicular to the \( x \)-axis.

\[
V_i = A_i \cdot \Delta x
\]

\[
A_i = \frac{1}{2} \cdot b_i \cdot h_i
\]

\[
A_i = \frac{1}{2} \cdot \frac{x}{4} \cdot \frac{x}{8} \sqrt{3}
\]

\[
= \frac{x^2}{64} \sqrt{3}
\]

\[
V_i = \frac{x^2}{64} \sqrt{3} \cdot \Delta x
\]

\[
V \approx \sum_{i=1}^{n} \frac{x^2}{64} \sqrt{3} \Delta x
\]

\[
V = \int_{0}^{20} \frac{\sqrt{3}}{64} x^2 \, dx = \frac{\sqrt{3}}{64} \frac{x^3}{3} \bigg|_{0}^{20} = \frac{\sqrt{3}}{64} \cdot \frac{20^3}{3} = \frac{125 \sqrt{3}}{3} \text{ m}^3
\]